6th ITER International School Ahmedabad INDIA

Physics of Landau and Cyclotron Resonances : Current Generation and Free Energy Extraction

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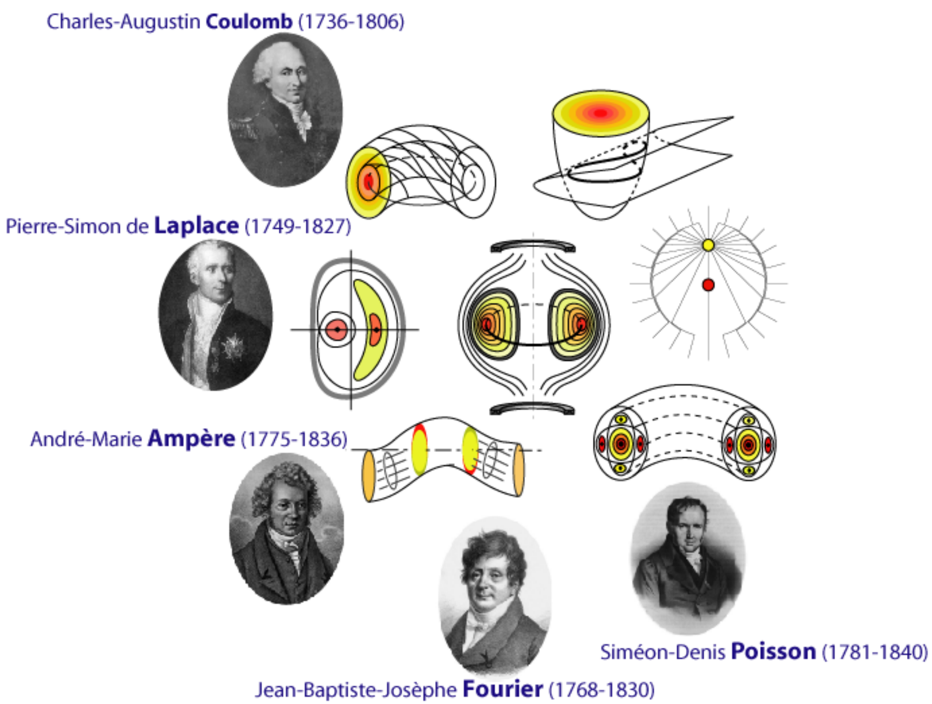

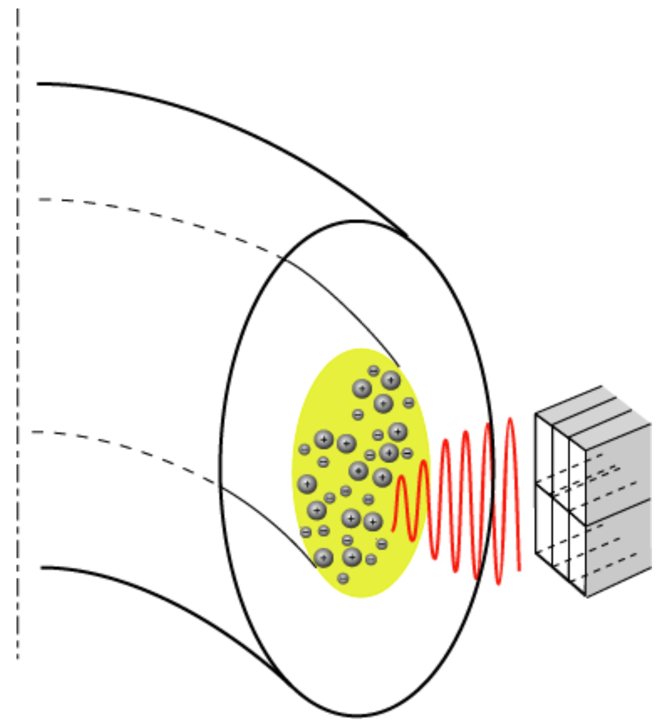

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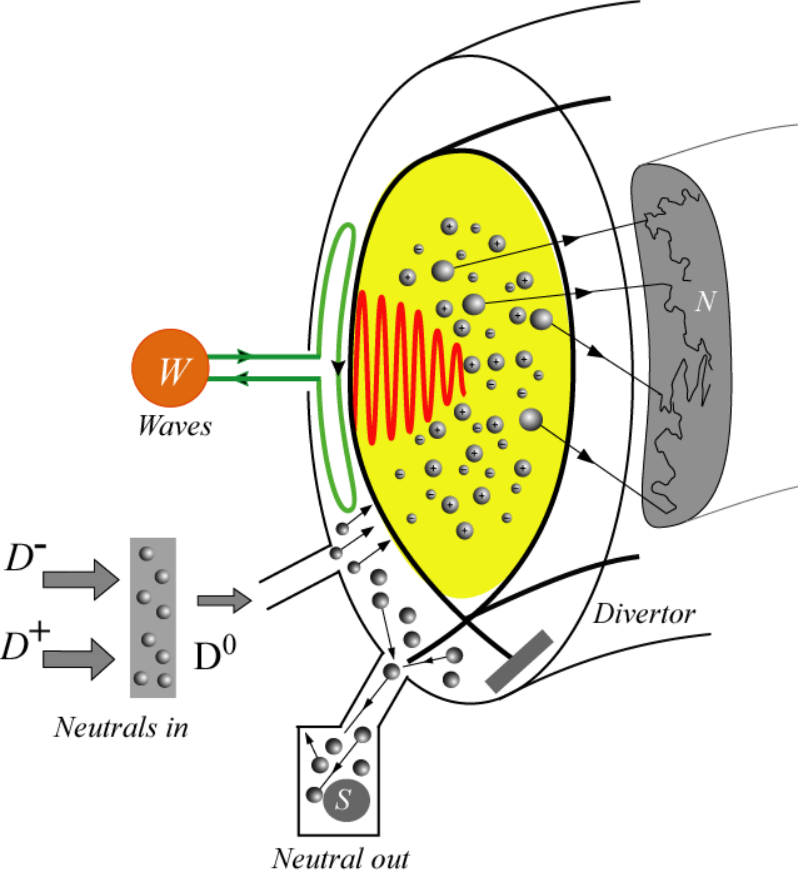

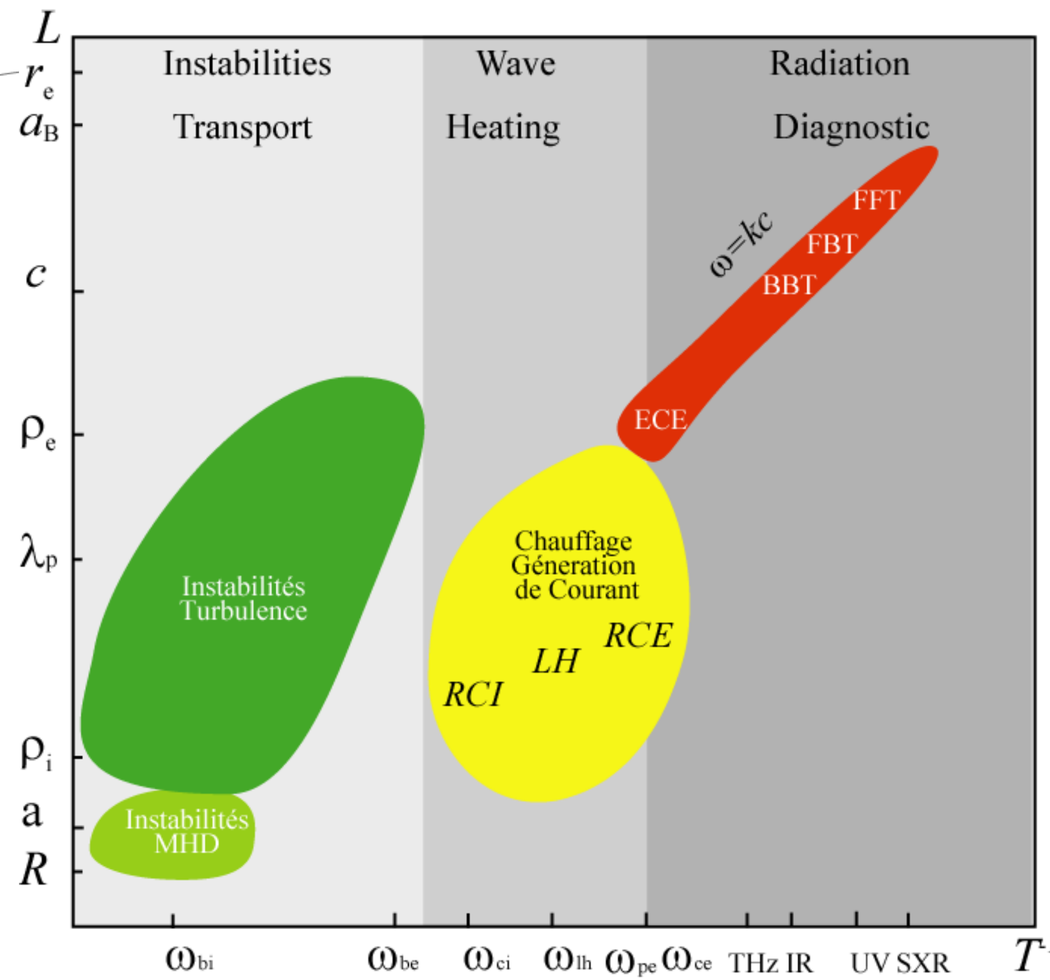

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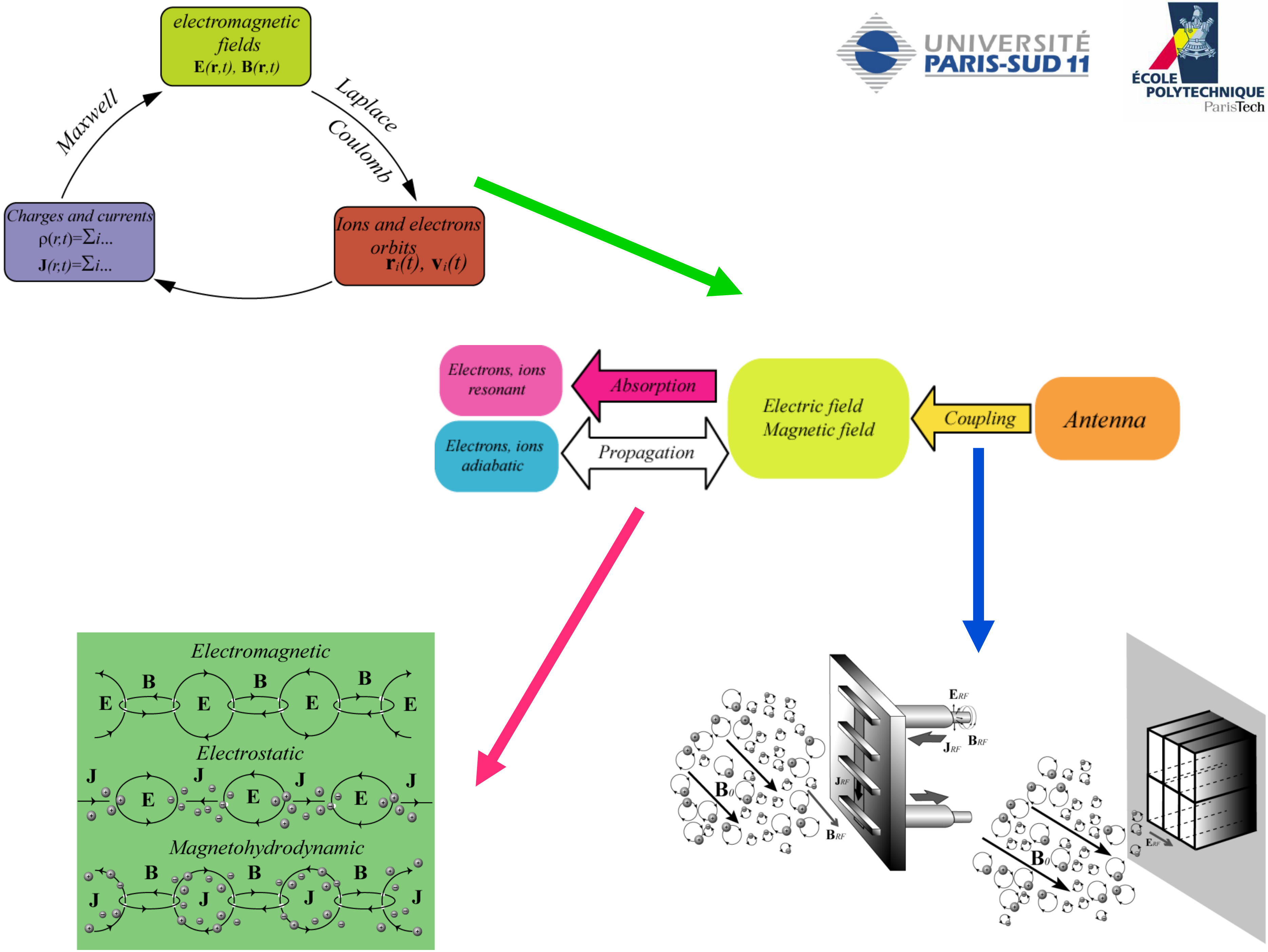
Physics of Landau and Cyclotron Resonances : Current Generation and Free Energy Extraction

- Active and reactive power
- Plasma resonances
- Resonant interaction
- Random phase approximation RPA
- Quasi linear equation
- Landau absorption
- Cyclotron absorption
- Current generation 1D
- Current generation 2D
- Free energy extraction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electromagnetic fields
E(r,t), B(r,t)
Laplace
Coulomb
Maxwell
Charges and currents
ρ(r,t)=Σi...
J(r,t)=Σi...
Ions and electrons orbits
r_i(t), v_i(t)
Electrons, ions resonant
Absorption
Electrons, ions adiabatic
Propagation
Electric field
Magnetic field
Coupling
Antenna
Electromagnetic
Electrostatic
Magnetohydrodynamic
B
E
J
B0
E_RF
B_RF
J_RF

Reactive power exchange

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{E}\exp -j\omega t \rightarrow \mathbf{v} = j\frac{q}{m\omega}\mathbf{E}\exp -j\omega t \rightarrow \langle \mathbf{v}\cdot\mathbf{E}\rangle_t = 0$$

Active power exchange

$$\frac{d\mathbf{v}}{dt} = -\nu\mathbf{v} + \frac{q}{m}\mathbf{E}\exp -j\omega t \rightarrow \langle \mathbf{v}\cdot\mathbf{E}\rangle_t \neq 0$$

$$k_{\parallel}v_{\parallel} - \omega \ll 0$$

$$k_{\parallel}v_{\parallel} - \omega \gg 0$$

Resonant collisionless power exchange

$$k_{\parallel}v_{\parallel} - \omega \approx 0$$

$$\frac{dv_{\parallel}}{dt} = \frac{q}{m}E\exp j\left(k_{\parallel}z - \omega t\right) \rightarrow \frac{dv_{\parallel}}{dt} = \frac{q}{m}E\exp j\left(k_{\parallel}v_{\parallel}t - \omega t\right)$$

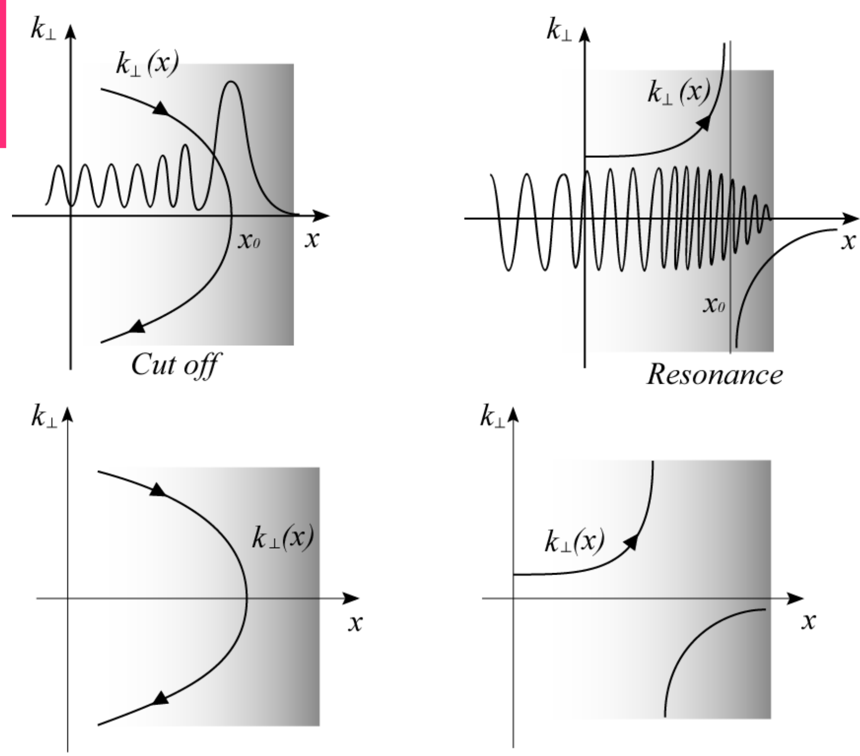
$$\frac{dv_{\parallel}}{dt} = \frac{q}{m}E\exp j\left(k_{\parallel}v_{\parallel}t - \omega t\right) \rightarrow v_{\parallel} = -j\frac{q}{m}\frac{E}{k_{\parallel}v_{\parallel} - \omega}\exp j\left(k_{\parallel}v_{\parallel}t - \omega t\right)$$

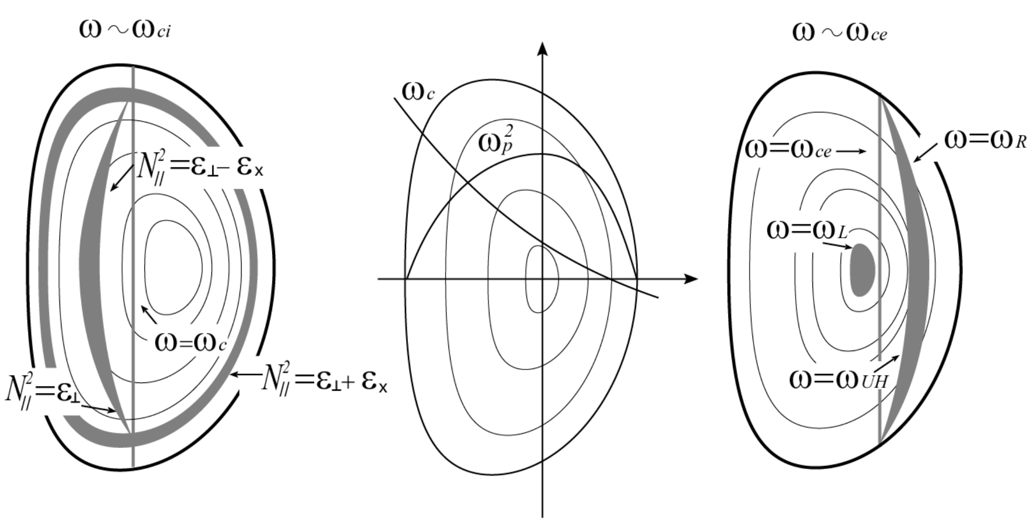
Cold Plasma Resonances

$$\begin{vmatrix} \varepsilon_\perp - N_\parallel^2 & -j\varepsilon_\times & N_\perp N_\parallel \\ j\varepsilon_\times & \varepsilon_\perp - N_\parallel^2 - N_\perp^2 & 0 \\ N_\perp N_\parallel & 0 & \varepsilon_\parallel - N_\perp^2 \end{vmatrix} = 0$$

D	$aN_\perp^4 - bN_\perp^2 + c = 0$
a	$\varepsilon_\perp$
b	$\left(\varepsilon_\perp + \varepsilon_\parallel\right)\left(\varepsilon_\perp - N_\parallel^2\right) - \varepsilon_\times^2$
c	$\varepsilon_\parallel\left(\varepsilon_\perp - N_\parallel^2\right)\left(\varepsilon_\perp - N_\parallel^2\right) - \varepsilon_\parallel\varepsilon_\times^2$

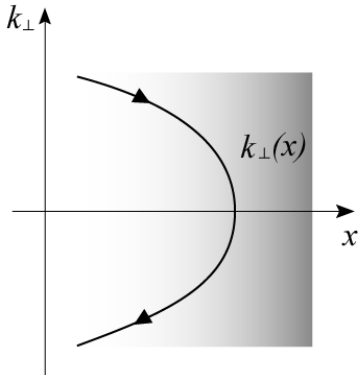
Resonances : $a = 0$	Cut off : $c = 0$
Absorption and Conversion	Evanescence and Reflection
Conversion of active power	Localization of reactive power

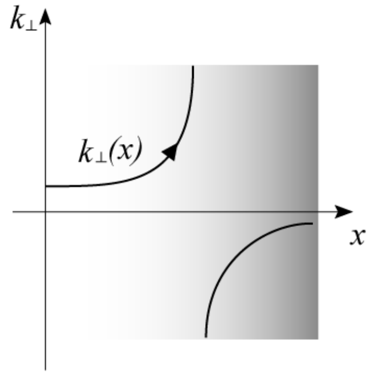




Cold Plasma

Cut Off



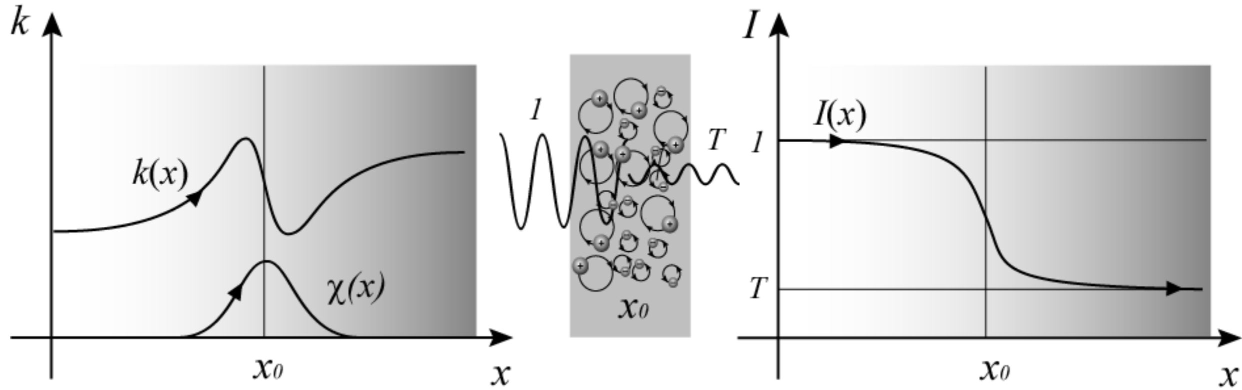


Resonance

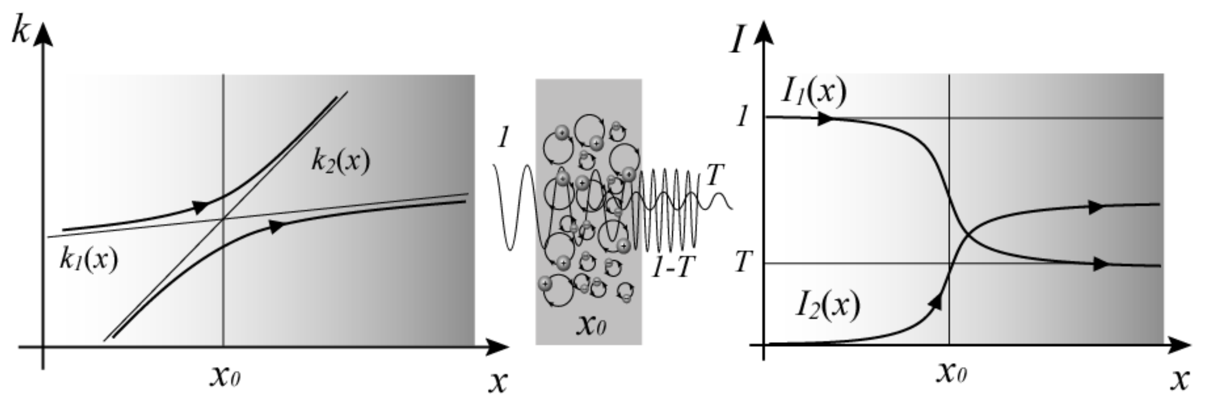
Kinetic effects

Hot Plasma

Resonant Absorption

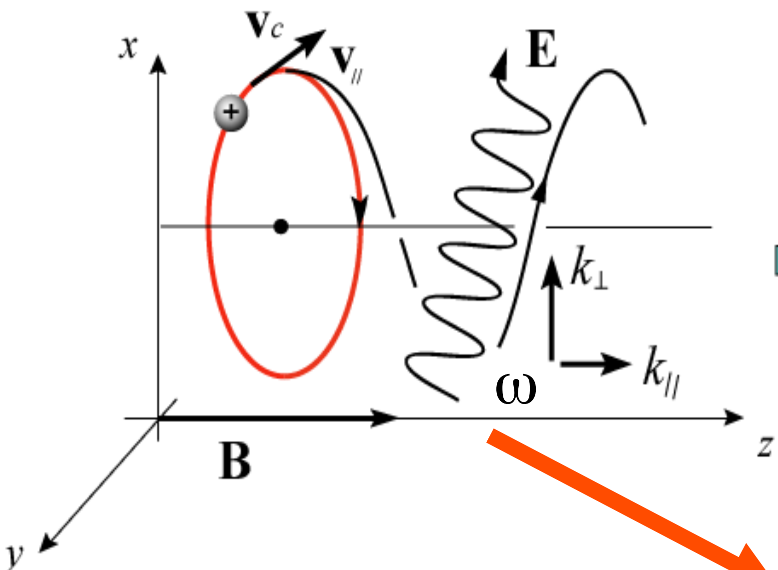


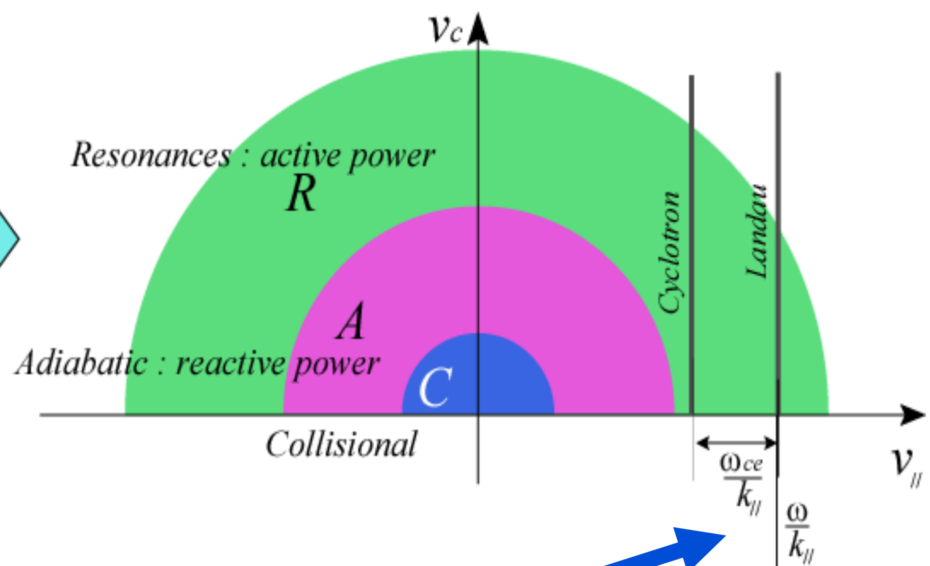
Mode Conversion



Resonant Absorption

Waves - Particles Resonances





Landau Resonances $\omega = k_{||} v_{||}$

Cyclotron Resonances $\omega - k_{||} v_{||} = n\omega_c$

Resonant interaction

$$Newton/Coulomb\ :\ m\frac{d^2z}{dt^2} = -\frac{\partial}{\partial z} q\phi \cos\left(kz - \omega_0 t\right)$$

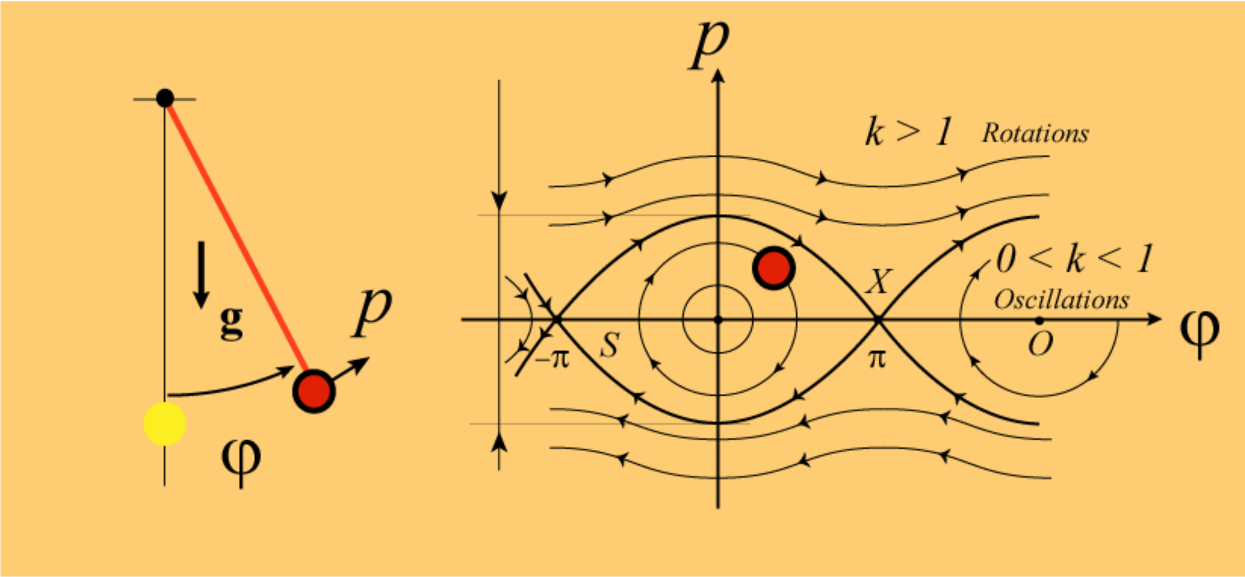
$$[z, dz/dt] \rightarrow [\varphi = kz - \omega_0 t,\ I = kv, \qquad \Omega^2 = qk^2\phi/m]$$

$$\begin{aligned} \frac{d\varphi}{dt} &= I - \omega_0 \\ \frac{dI}{dt} &= -\frac{\partial}{\partial \varphi}\Omega^2 \cos\varphi \end{aligned}$$

$$\frac{d\varphi}{dt} = I - \omega_0$$
$$\frac{dI}{dt} = -\frac{\partial}{\partial\varphi}\Omega^2 \cos\varphi$$

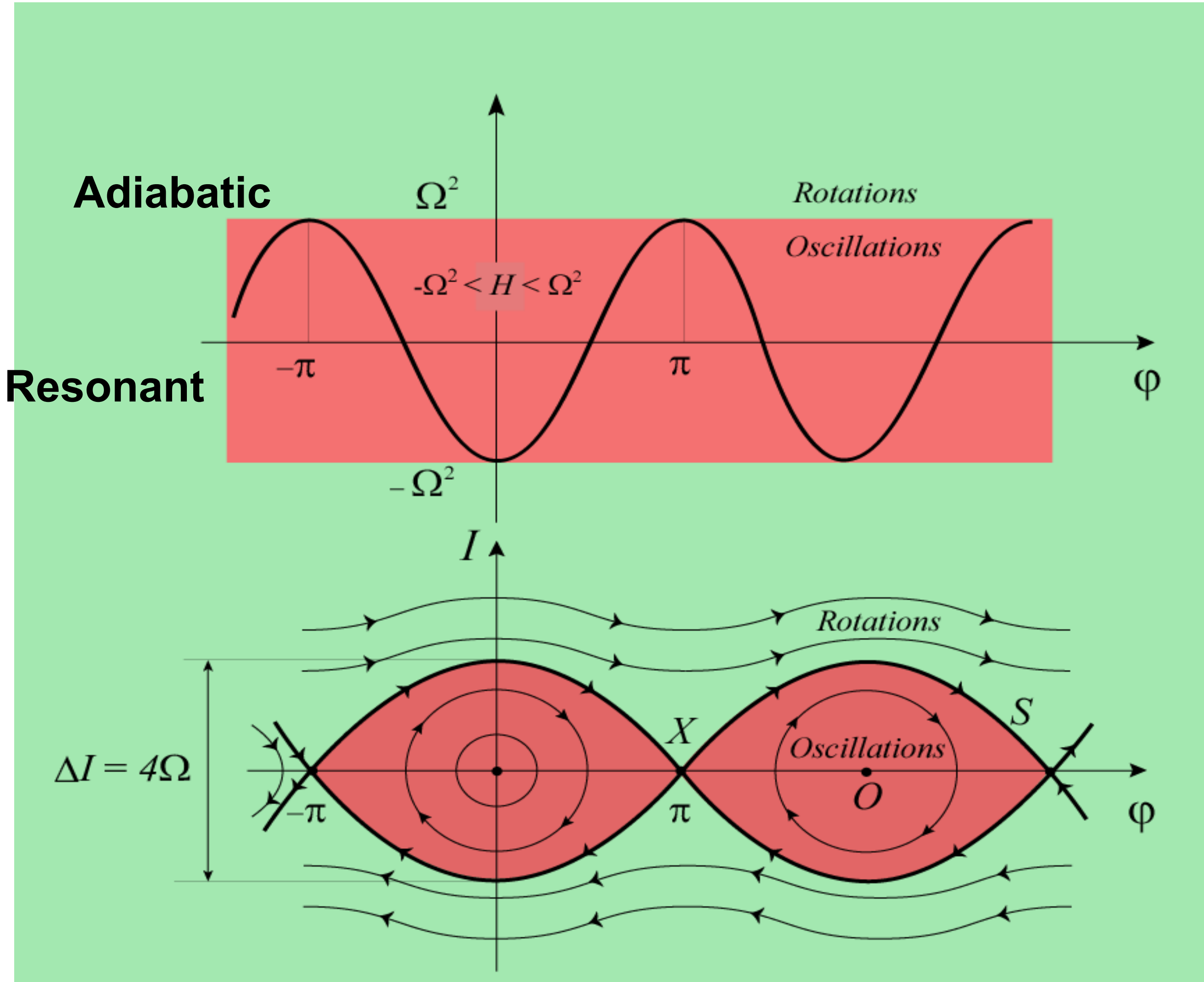
$$Potential : E_p = \Omega^2 \cos\varphi \; , \; Kinetic : E_c = \frac{(I-\omega_0)^2}{2}$$

Nonlinear pendulum



Nonlinear pendulum

$$Potential : E_p = \Omega^2 \cos\varphi \ , \ Kinetic : E_c = \frac{(I - \omega_0)^2}{2}$$



$$(I-\omega_0)^2 - 2\Omega^2\cos\varphi = (I_0-\omega_0)^2 - 2\Omega^2\cos\varphi_0 \quad \longleftrightarrow \quad \begin{aligned} \frac{d\varphi}{dt} &= I-\omega_0 \\ \frac{dI}{dt} &= -\frac{\partial}{\partial\varphi}\Omega^2\cos\varphi \end{aligned}$$

1st order

$$I(I_0,\varphi_0,t) - I_0 = -\frac{2\Omega^2}{I_0-\omega_0}\sin\left(\frac{I_0-\omega_0}{2}t\right)\sin\left(\frac{I_0-\omega_0}{2}t+\varphi_0\right)$$

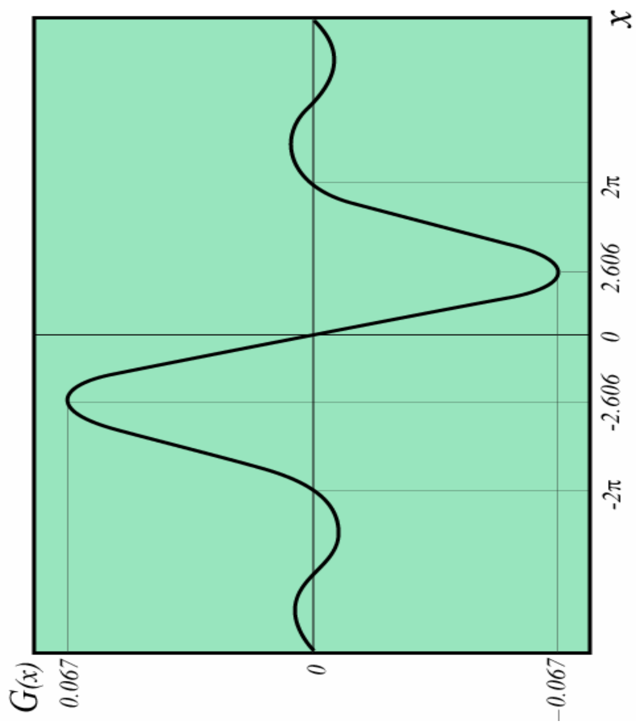
$$\frac{1}{2\pi}\oint (I-I_0)\,d\varphi_0 = \langle\delta I\rangle_{\varphi_0} = 0$$

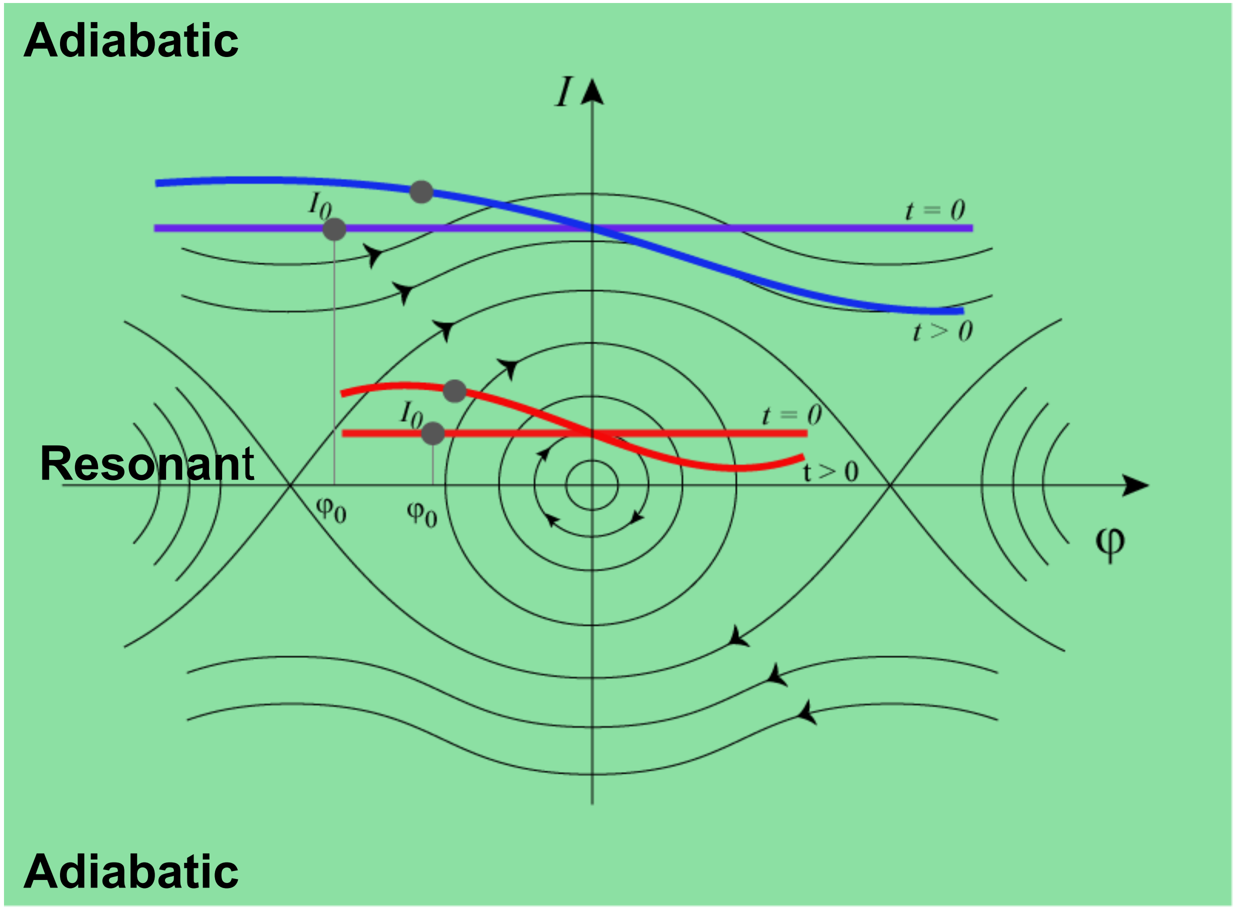
$$\frac{1}{2\pi}\oint (I-I_0)^2\,d\varphi_0 = \langle\delta I^2\rangle_{\varphi_0} = \frac{2\Omega^4}{(I_0-\omega_0)^2}\sin^2\left(\frac{I_0-\omega_0}{2}t\right)$$

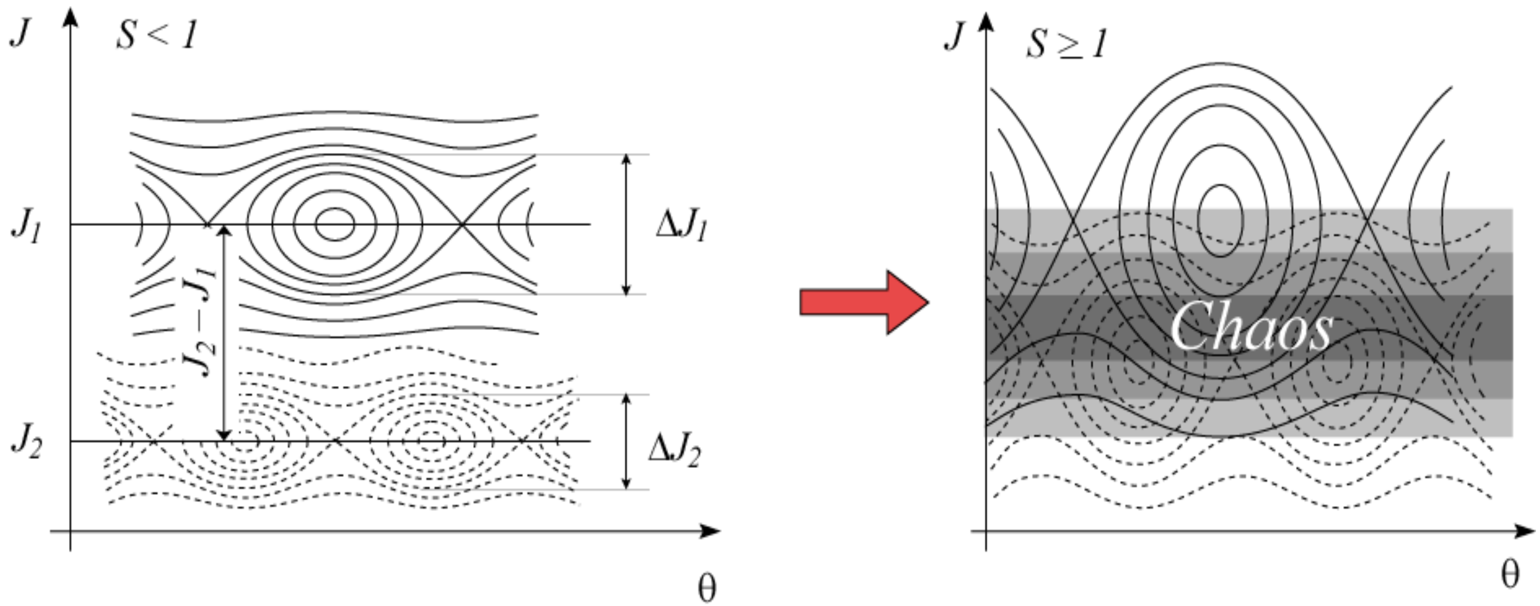
2nd order

$$\langle\delta I\rangle_{\varphi_0} = \Omega^4\frac{\partial}{\partial I_0}\frac{\sin^2\left(\frac{I_0-\omega_0}{2}t\right)}{(I_0-\omega_0)^2} \equiv \Omega^4 G\left[(I_0-\omega_0)\,t\right]t^2$$

Energy Transfer between wave and particle : G







Resonance

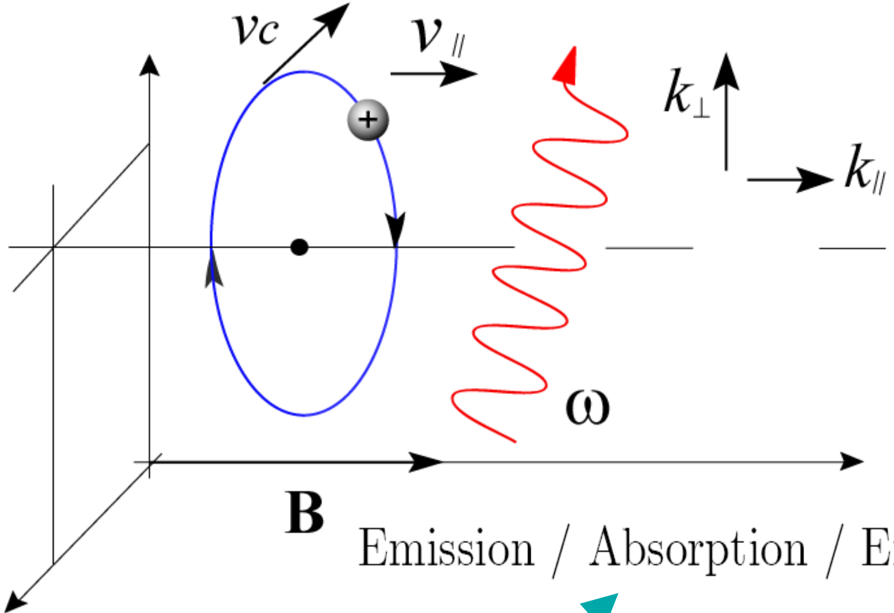
Resonances

Chaos

Asymptotic model → **Random Walk** **RPA**

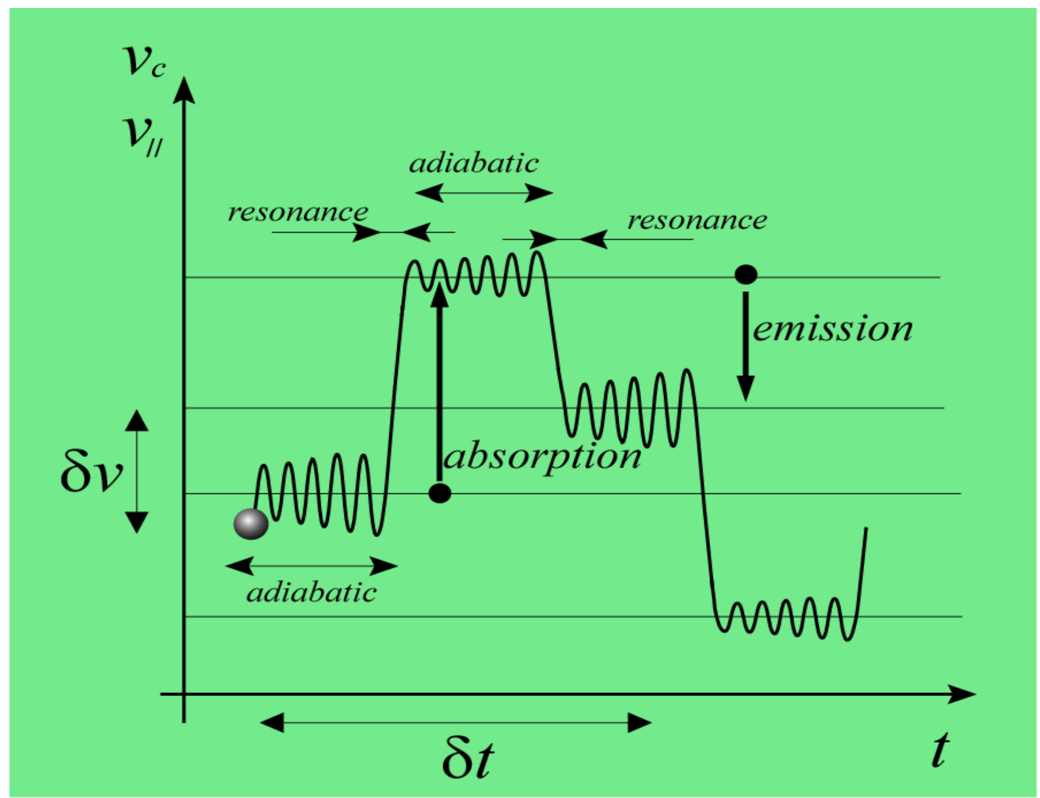
Emission / Absorption / Emission /Emission / Absorption / Emission...

Quasilinear theory

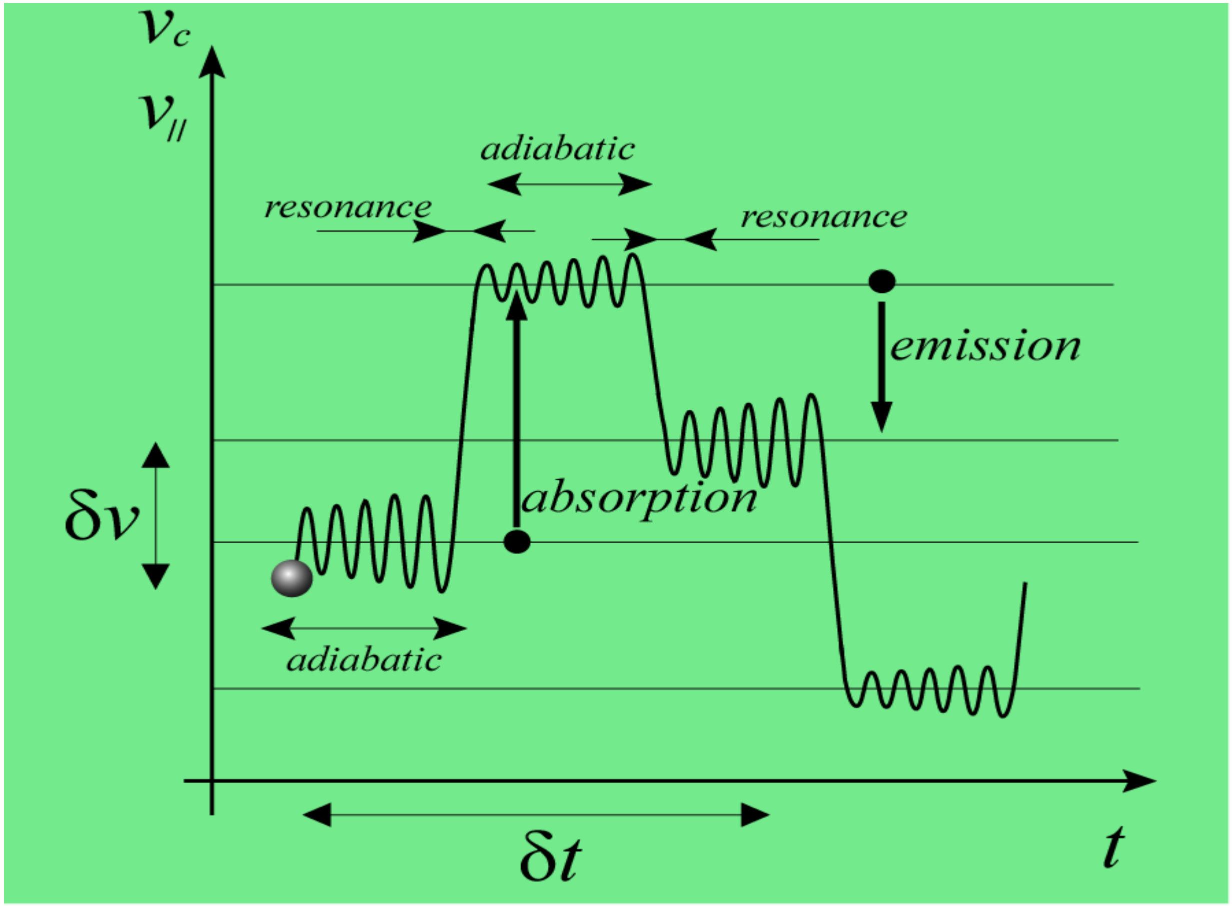


Emission / Absorption / Emission /Emission / Absorption / Emission...

RPA



Emission / Absorption / Emission /Emission / Absorption / Emission...



RPA

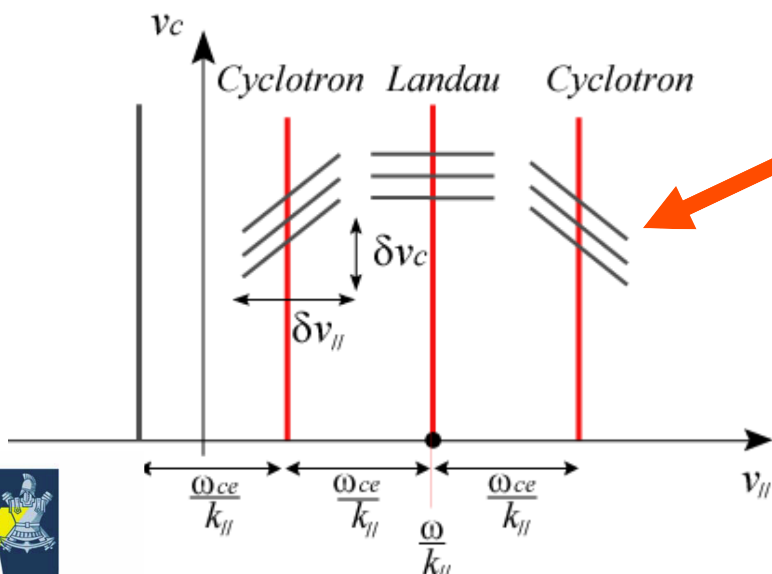
Quasi-linear equation :
$$\frac{\partial f(\mathbf{v},t)}{\partial t} = \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta\mathbf{v}\delta\mathbf{v}\rangle_{\mathbf{x}_0}}{2\delta t} \cdot \frac{\partial}{\partial \mathbf{v}} f(\mathbf{v},t)$$

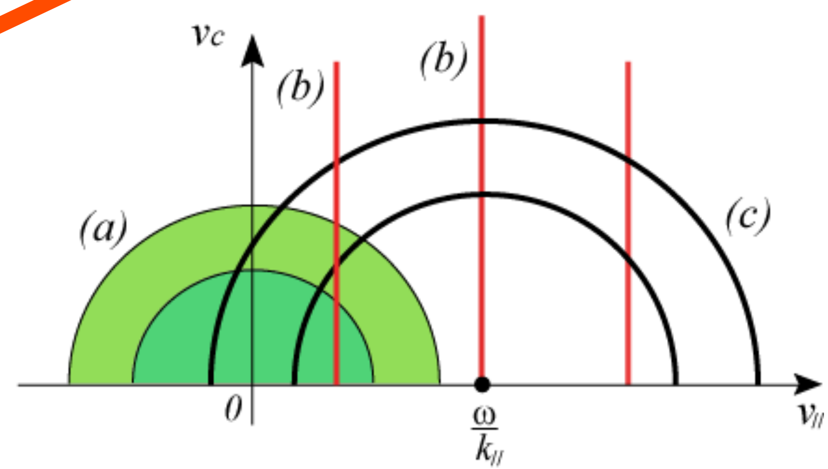
Emission / Absorption / Emission /Emission / Absorption / Emission...

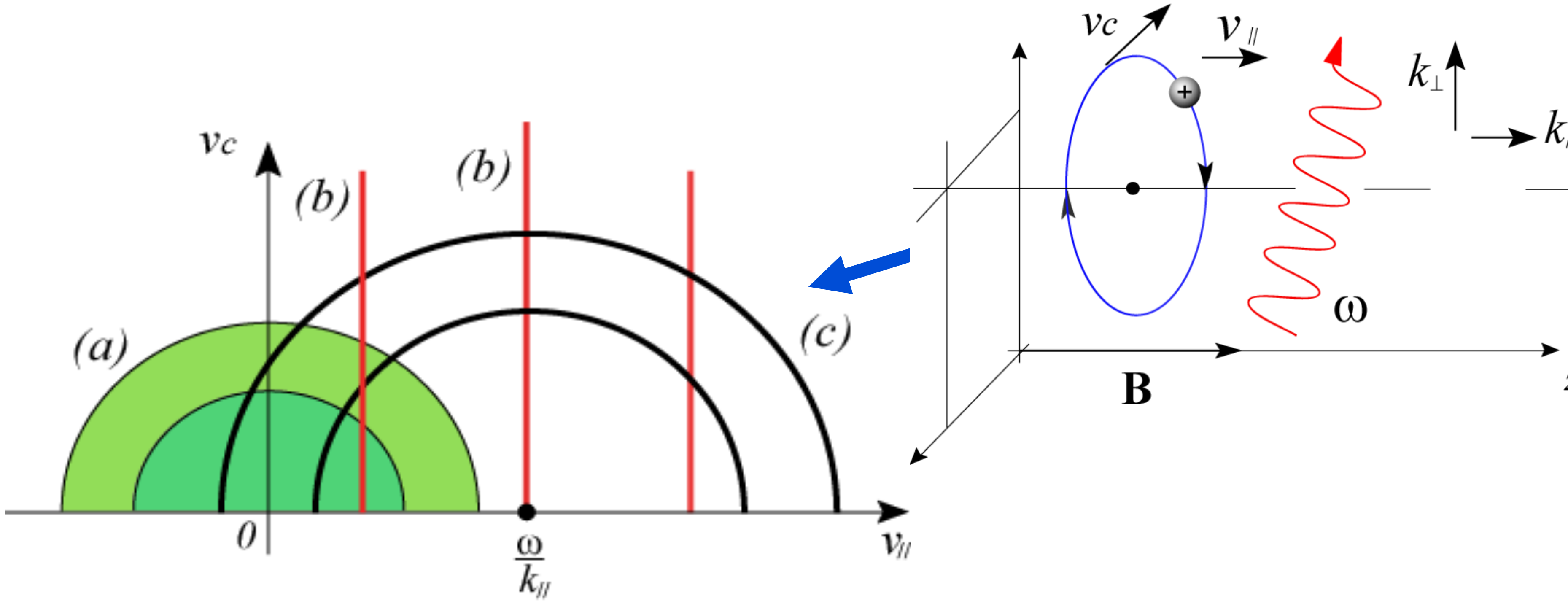
$$\delta\left(\frac{m}{2}v^2\right) = \hbar\omega,\ \delta\left(mv_\parallel\right) = \hbar k_\parallel,\ \delta\left(\frac{m}{2}v_c^2\right) = N\hbar\omega_c$$

$$N\omega_c + k_\parallel v_\parallel = \omega$$

$$\frac{N\omega_c}{k_\parallel} \rightarrow \frac{\delta v_c}{\delta v_\parallel} = \frac{N\omega_c}{k_\parallel v_c} \underset{N\omega_c + k_\parallel v_\parallel = \omega}{\longrightarrow} \frac{\delta v_c}{\delta v_\parallel} = \frac{\omega - k_\parallel v_\parallel}{k_\parallel v_c} = -\frac{v_\parallel - \frac{\omega}{k_\parallel}}{v_c}$$





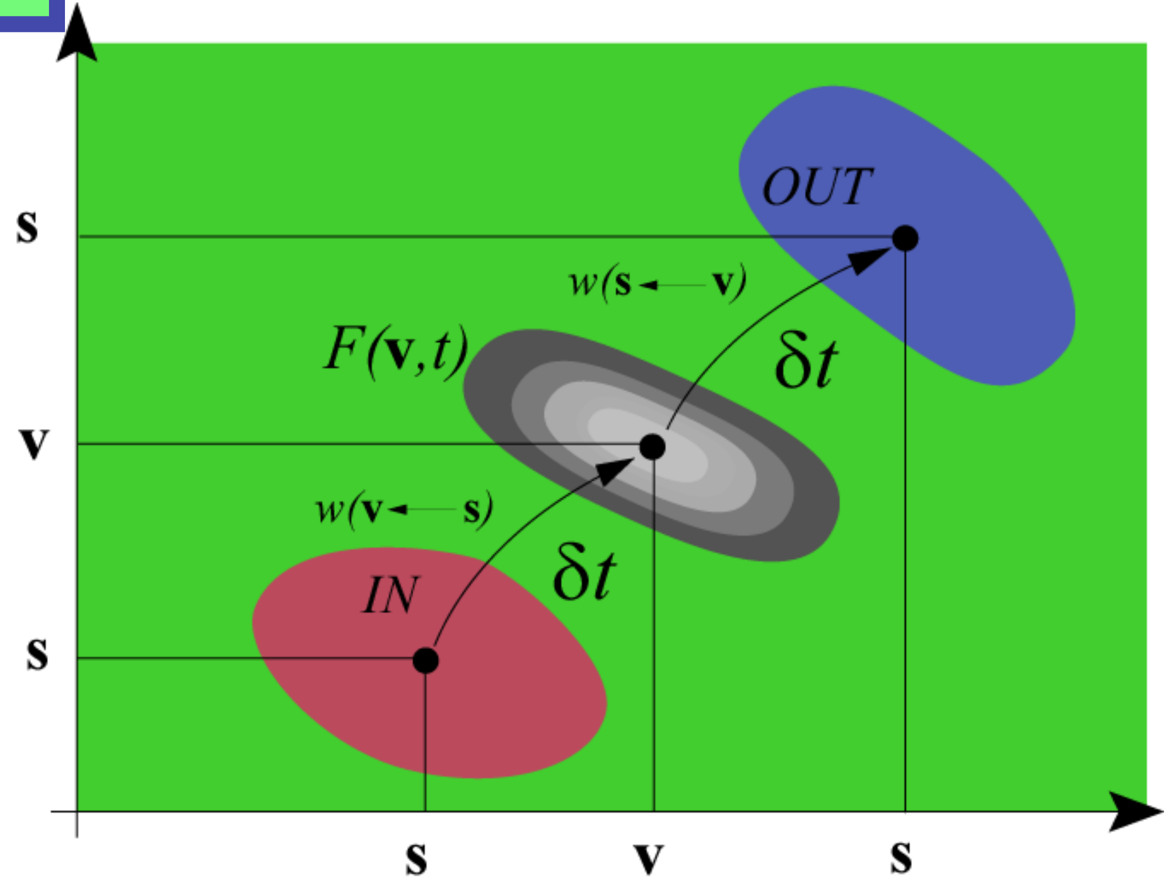


Resonance curves (b) : $\omega = N\omega_c + k_\parallel v_\parallel$

Isoenergy lines (a) : $H = \frac{m}{2}v_c^2 + \frac{m}{2}v_\parallel^2$

Diffusion path (c) : $v_c^2 + \left(v_\parallel - \frac{\omega}{k_\parallel}\right)^2 = v_{0c}^2 + \left(v_{0\parallel} - \frac{\omega}{k_\parallel}\right)^2$

Quasi linear equation



$$\delta F(\mathbf{v},t) = \delta t \int w(\mathbf{v} \leftarrow \mathbf{s}) F(\mathbf{s},t)\, d\mathbf{s} - \delta t \int w(\mathbf{s} \leftarrow \mathbf{v}) F(\mathbf{v},t)\, d\mathbf{s}$$

$$\frac{\partial F(\mathbf{v},t)}{\partial t} = \int \left[w(\mathbf{v} \leftarrow \mathbf{v}+\mathbf{x}) F(\mathbf{v}+\mathbf{x},t) - w(\mathbf{v}+\mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v},t) \right] d\mathbf{x}$$

$$\frac{\partial F(\mathbf{v},t)}{\partial t} = \int \left[w(\mathbf{v} \leftarrow \mathbf{v}+\mathbf{x}) F(\mathbf{v}+\mathbf{x},t) - w(\mathbf{v}+\mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v},t) \right] d\mathbf{x}$$

$$\begin{aligned} w(\mathbf{v} \leftarrow \mathbf{v}+\mathbf{x}) F(\mathbf{v}+\mathbf{x}) &= w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v}) \\ &+ \mathbf{x} \cdot \frac{\partial w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v})}{\partial \mathbf{v}} \\ &+ \frac{\mathbf{x}\mathbf{x}}{2} \cdot \frac{\partial^2 w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) F(\mathbf{v})}{\partial \mathbf{v} \partial \mathbf{v}} \end{aligned}$$

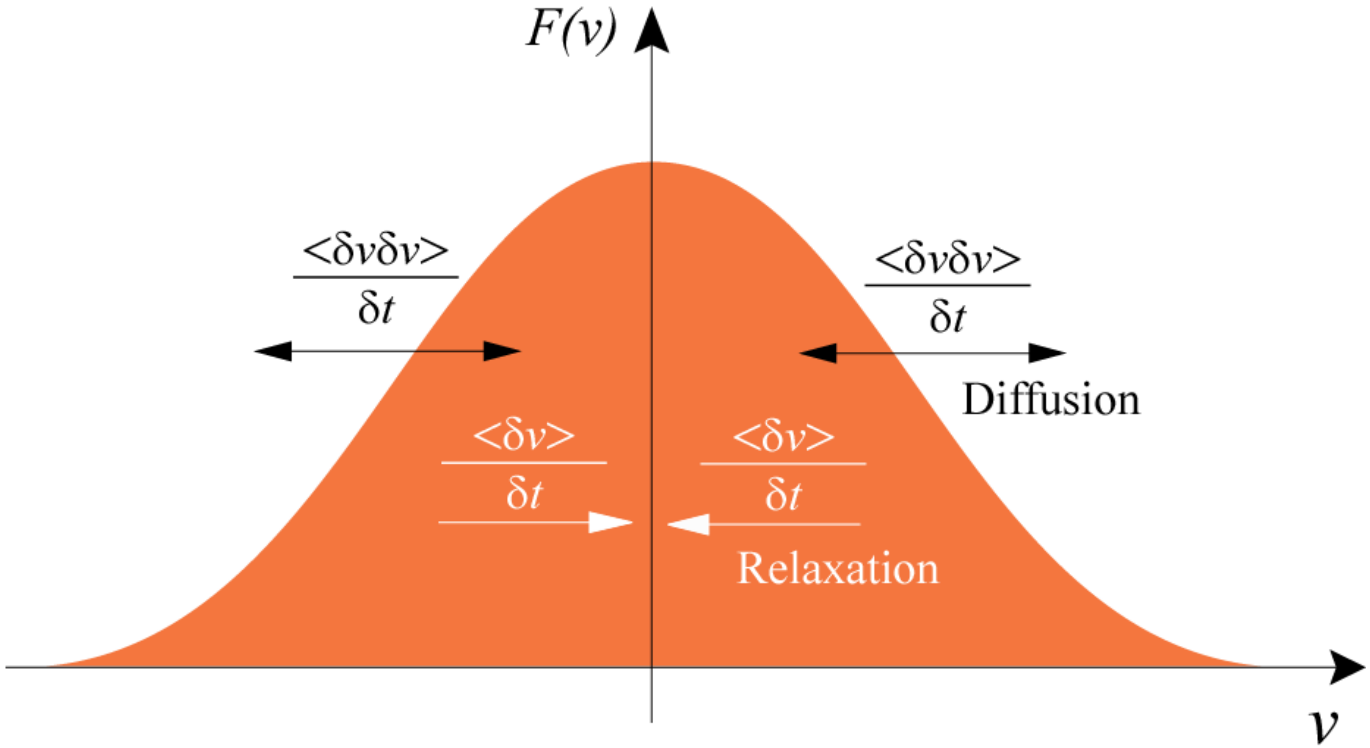
Taylor : Kramer-Moyal

$$\int d\mathbf{x} \left[w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) - w(\mathbf{v}+\mathbf{x} \leftarrow \mathbf{v}) \right] = 0$$

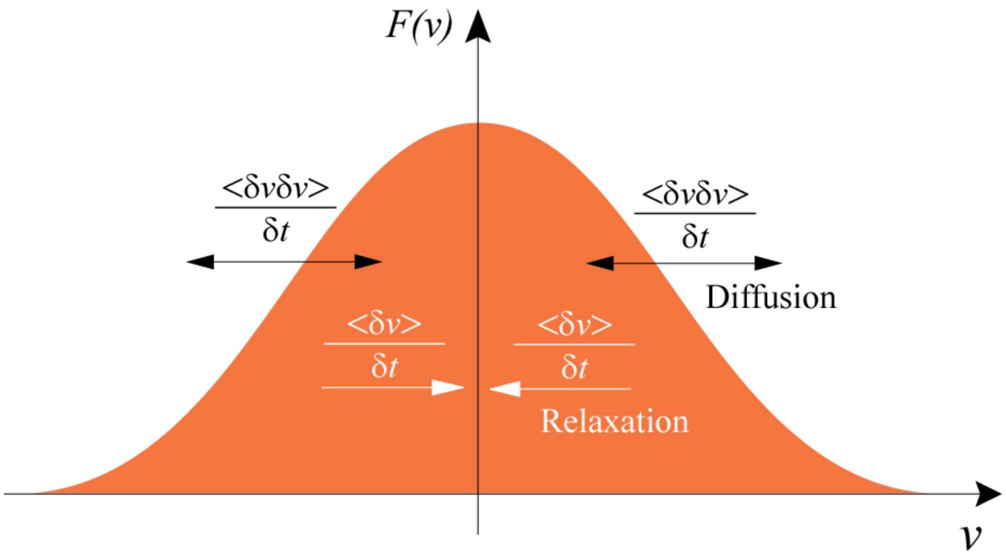
$$\mathit{Friction} \quad : \quad \frac{\langle \delta \mathbf{v} \rangle}{\delta t} \equiv -\int \mathbf{x} w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) \, d\mathbf{x} = \int \mathbf{x} w(\mathbf{v}+\mathbf{x} \leftarrow \mathbf{v}) \, d\mathbf{x}$$

$$\mathit{Diffusion} \quad : \quad \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{\delta t} \equiv \int \mathbf{x}\mathbf{x} w(\mathbf{v}-\mathbf{x} \leftarrow \mathbf{v}) \, d\mathbf{x} = \int \mathbf{x}\mathbf{x} w(\mathbf{v}+\mathbf{x} \leftarrow \mathbf{v}) \, d\mathbf{x}$$

$$\frac{\partial F(\mathbf{v},t)}{\partial t} = -\frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{\langle \delta \mathbf{v} \rangle}{\delta t} F(\mathbf{v},t) - \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{2 \delta t} F(\mathbf{v},t) \right]$$



$$\textit{Fokker-Planck} : \frac{\partial F(\mathbf{v},t)}{\partial t} = -\frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{\langle \delta \mathbf{v} \rangle}{\delta t} F(\mathbf{v},t) - \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{2\delta t} F(\mathbf{v},t) \right]$$



$$Microreversibility\ :\ w\left(\mathbf{v}+\mathbf{x}\leftarrow\mathbf{v}\right)=w\left(\mathbf{v}\leftarrow\mathbf{v}+\mathbf{x}\right)$$

$$w\left(\mathbf{v}\leftarrow\mathbf{v}+\mathbf{x}\right)=w\left(\mathbf{v}+\mathbf{x}\leftarrow\mathbf{v}\right)=w\left(\mathbf{v}-\mathbf{x}\leftarrow\mathbf{v}\right)+\mathbf{x}\cdot\frac{\partial w\left(\mathbf{v}-\mathbf{x}\leftarrow\mathbf{v}\right)}{\partial\mathbf{v}}$$

$$\begin{aligned}\frac{\langle\delta\mathbf{v}\rangle}{\delta t} &= \frac{1}{2}\int\mathbf{x}w\left(\mathbf{v}+\mathbf{x}\leftarrow\mathbf{v}\right)d\mathbf{x}-\frac{1}{2}\int\mathbf{x}w\left(\mathbf{v}-\mathbf{x}\leftarrow\mathbf{v}\right)d\mathbf{x}\\ &= \frac{1}{2}\int\mathbf{x}\mathbf{x}\cdot\frac{\partial w\left(\mathbf{v}-\mathbf{x}\leftarrow\mathbf{v}\right)}{\partial\mathbf{v}}d\mathbf{x}\end{aligned}$$

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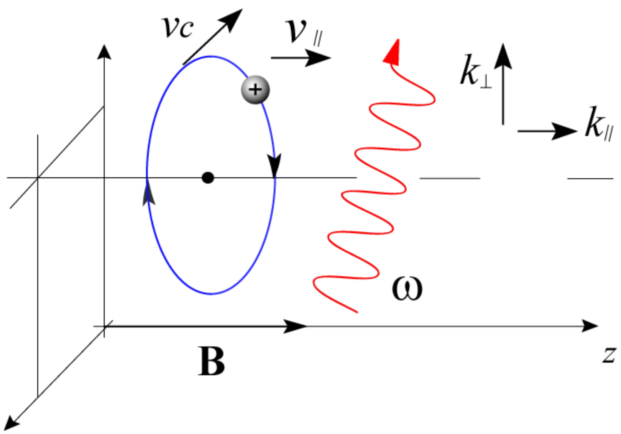
$$\textit{Fokker-Planck}: \frac{\partial F(\mathbf{v},t)}{\partial t} = -\frac{\partial}{\partial \mathbf{v}} \cdot \left[\frac{\langle \delta \mathbf{v} \rangle}{\delta t} F(\mathbf{v},t) - \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{2\delta t} F(\mathbf{v},t) \right]$$

$$\textit{Einstein Relation}: \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{2\delta t} = \frac{\langle \delta \mathbf{v} \rangle}{\delta t}$$

$$\textit{Quasi linear equation}: \frac{\partial F}{\partial t} = \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta \mathbf{v} \delta \mathbf{v} \rangle}{2\delta t} \cdot \frac{\partial F}{\partial \mathbf{v}}$$

$$\textit{Quasi-linear equation} : \frac{\partial f(\mathbf{v},t)}{\partial t} = \frac{\partial}{\partial \mathbf{v}} \cdot \frac{\langle \delta\mathbf{v}\delta\mathbf{v} \rangle_{\mathbf{x}_0}}{2\delta t} \cdot \frac{\partial}{\partial \mathbf{v}} f(\mathbf{v},t)$$

RPA



$$\textit{Landau} : \frac{\partial f(v_{\|},t)}{\partial t} = \frac{\partial}{\partial v_{\|}} \frac{\langle \delta v_{\|} \delta v_{\|} \rangle}{2\delta t} \frac{\partial f}{\partial v_{\|}}$$

$$\textit{Cyclotron} : \frac{\partial f(v_c,t)}{\partial t} = \frac{1}{v_c} \frac{\partial}{\partial v_c} v_c \frac{\langle \delta v_c \delta v_c \rangle}{2\delta t} \frac{\partial f}{\partial v_c}$$

$$\textit{Collisions} : \frac{\partial f(v,t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} v^2 \frac{\langle \delta v \delta v \rangle}{2\delta t} \frac{\partial f}{\partial v}$$

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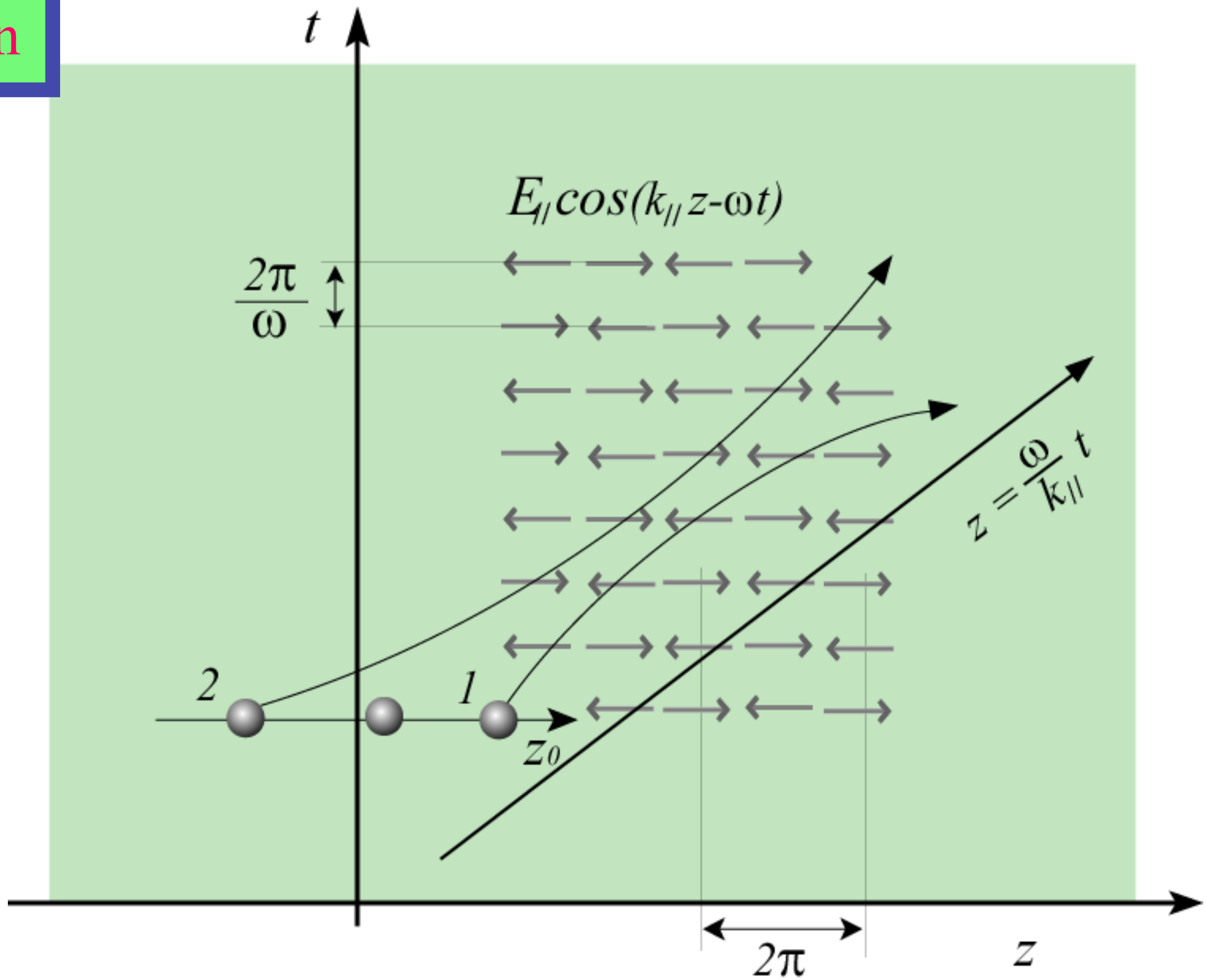
Landau absorption



Emission / Absorption / Emission /Emission / Absorption / Emission...

$$\frac{\partial f(v_\parallel,t)}{\partial t} = \frac{\partial}{\partial v_\parallel} \frac{\langle \delta v_\parallel \delta v_\parallel \rangle}{2\delta t} \frac{\partial f(v_\parallel,t)}{\partial v_\parallel}$$

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$$\frac{\partial f(v_\parallel,t)}{\partial t} = \frac{\partial}{\partial v_\parallel}\frac{\langle \delta v_\parallel \delta v_\parallel\rangle}{2\delta t}\frac{\partial f(v_\parallel,t)}{\partial v_\parallel}$$

$$w_L = n\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\frac{m}{2}v_\parallel^2 f dv_\parallel \rightarrow w_L = -nm\int_{-\infty}^{+\infty} v_\parallel \frac{\langle \delta v_\parallel \delta v_\parallel\rangle}{2\delta t}\frac{\partial f}{\partial v_\parallel}dv_\parallel$$

$$Newton/Coulomb : \frac{dv_\parallel}{dt} = -\frac{q}{m}\frac{d}{dz}\phi\cos\left(k_\parallel z - \omega t\right)$$

$$v' = v_\parallel - \omega/k_\parallel$$

$$mv'^2 + 2e\phi\cos k_\parallel z' = mv_0'^2 + 2e\phi\cos k_\parallel z_0' \rightarrow m\left(v' - v_0'\right)\left(v' + v_0'\right) = -2e\phi\left(\cos k_\parallel z' - \cos k_\parallel z_0'\right)$$

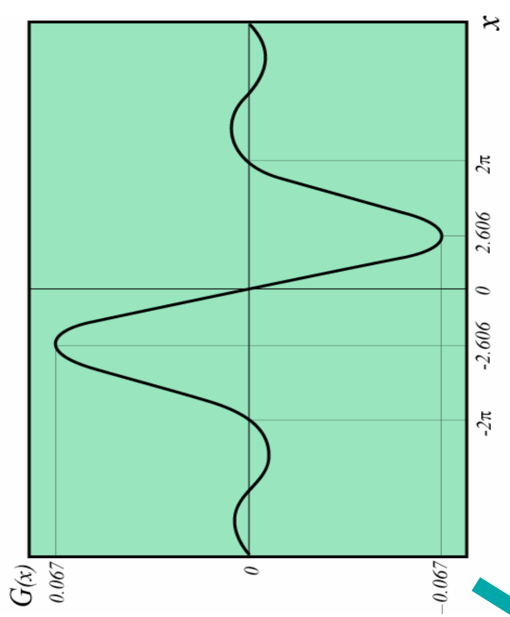
$$\delta v' = \frac{2e\phi}{mv_0'}\sin\left(k_\parallel\frac{v_0'\delta t}{2} + k_\parallel z_0'\right)\sin\left(k_\parallel\frac{v_0'\delta t}{2}\right)$$

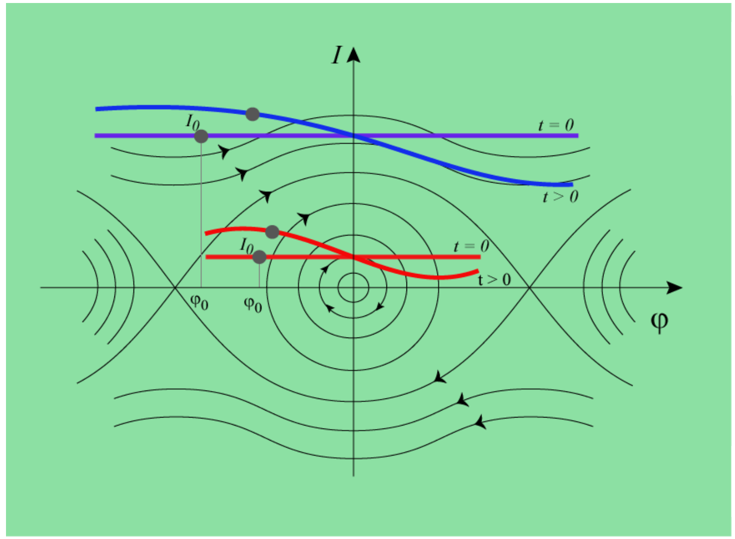
$$v' = v_\parallel - \omega/k_\parallel$$

$$\delta v_\parallel\left(v_{\parallel 0},\delta t\right) = -\frac{2e\phi}{m\left(v_{\parallel 0} - \omega/k_\parallel\right)}\sin\left[\frac{k_\parallel\left(v_{\parallel 0} - \omega/k_\parallel\right)\delta t}{2} + k_\parallel z_0\right]\sin\left[\frac{k_\parallel\left(v_{\parallel 0} - \omega/k_\parallel\right)\delta t}{2}\right]$$

$$\varepsilon\sin^2\left(x/\varepsilon\right)/x^2\Big|_{\varepsilon\to 0} \rightarrow \pi\delta\left(x\right)$$

$$\left\langle \delta v_\parallel^2\left(v_{\parallel 0}\right)\right\rangle_{z_0} = \frac{2e^2\phi^2}{m^2\left(v_{\parallel 0} - \omega/k_\parallel\right)^2}\sin^2\left[k_\parallel\frac{\left(v_{\parallel 0} - \omega/k_\parallel\right)\delta t}{2}\right] \rightarrow \pi\frac{e^2k_\parallel^2\phi^2}{m^2}\delta\left(k_\parallel v_{\parallel 0} - \omega\right)\delta t$$





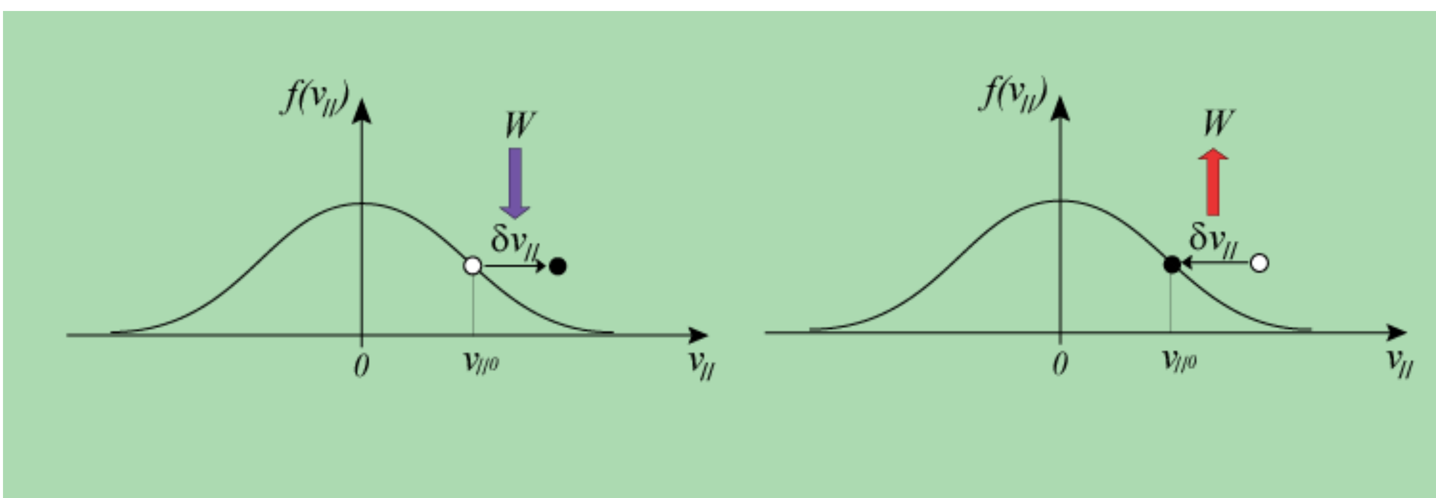
$$k_\parallel v_\parallel - \omega \gg$$

$$k_\parallel v_\parallel - \omega \approx 0$$

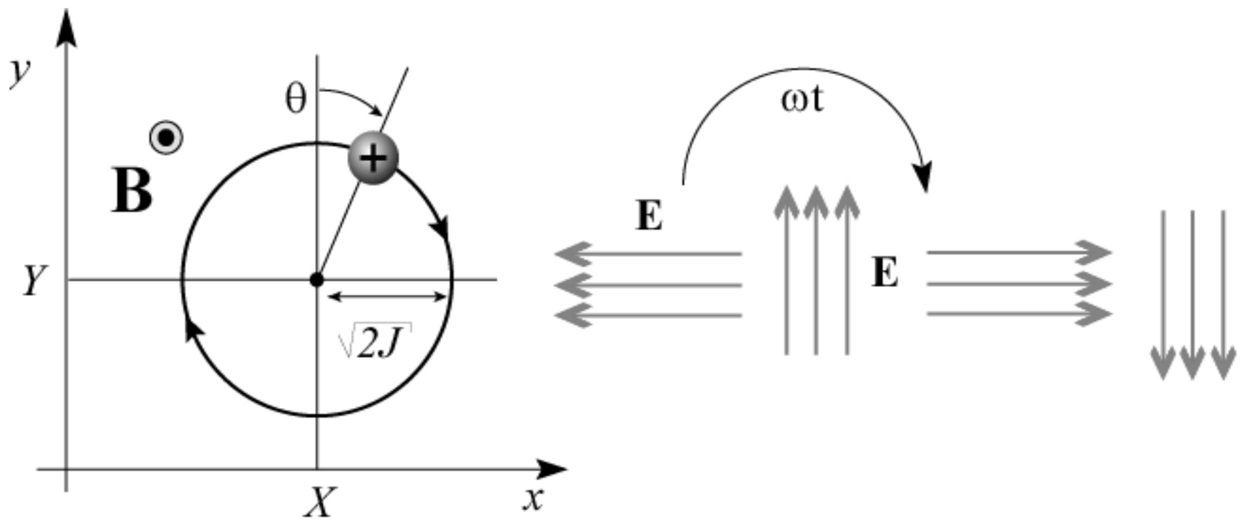
$$k_\parallel v_\parallel - \omega \ll 0$$

$$\varepsilon \sin^2 (x/\varepsilon)/x^2 \big|_{\varepsilon \to 0} \to \pi \delta(x)$$

$$Landau: \quad w_L = -\pi \omega_p^2 \frac{\omega}{k_\parallel} \frac{\partial f}{k_\parallel \partial v_\parallel}\bigg|_{k_\parallel v_\parallel = \omega} \frac{\varepsilon_0 E_\parallel^2}{2}$$



Cyclotron absorption



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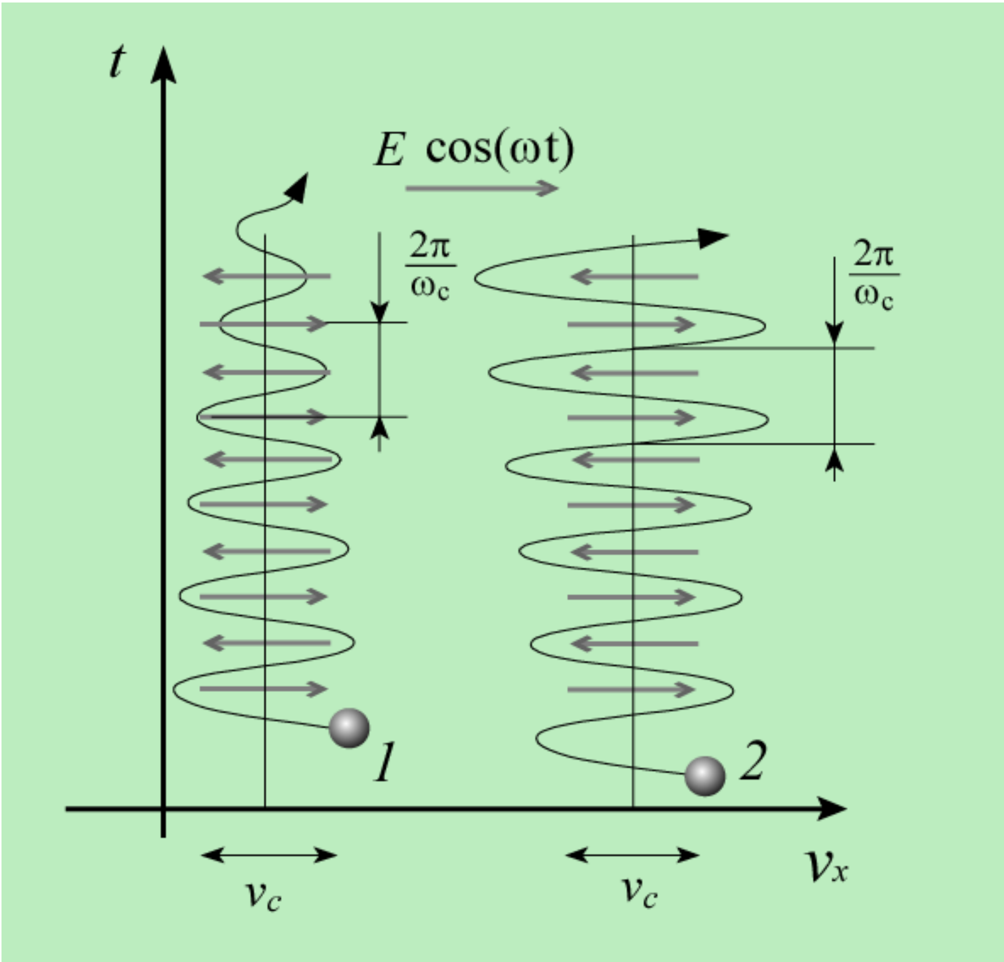
Emission / Absorption / Emission /Emission / Absorption / Emission...



$$\frac{\partial f(v_c,t)}{\partial t} = \frac{1}{v_c}\frac{\partial}{\partial v_c} v_c \frac{\langle \delta v_c \delta v_c \rangle}{2\delta t}\frac{\partial f(v_c,t)}{\partial v_c}$$

$$w_c = n\frac{\partial}{\partial t}\int_0^{+\infty} 2\pi v_c \frac{m v_c^2}{2} f\, dv_c \rightarrow w_c = -n\int_0^{+\infty} 2\pi m v_c^2 \frac{\langle \delta v_c \delta v_c \rangle}{2\delta t}\frac{\partial f}{\partial v_c} dv_c$$

$$Newton/Laplace/Coulomb:\ \frac{d\mathcal{Z}}{dt}+j\omega_c\mathcal{Z}=\frac{eE}{m}\exp -j\omega t$$

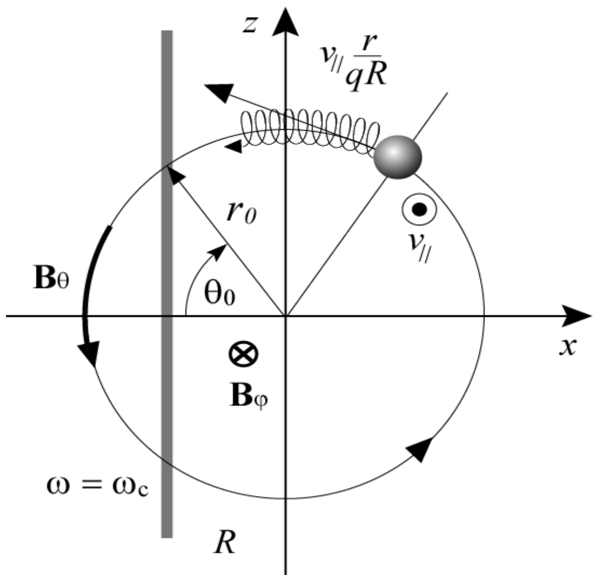
$$\mathcal{Z}(t)=v_c\exp(-j\omega_c t)+\frac{eE}{m}\exp(-j\omega_c t)\int_0^t \exp(j\omega_c u)\exp(-j\omega u)\,du$$

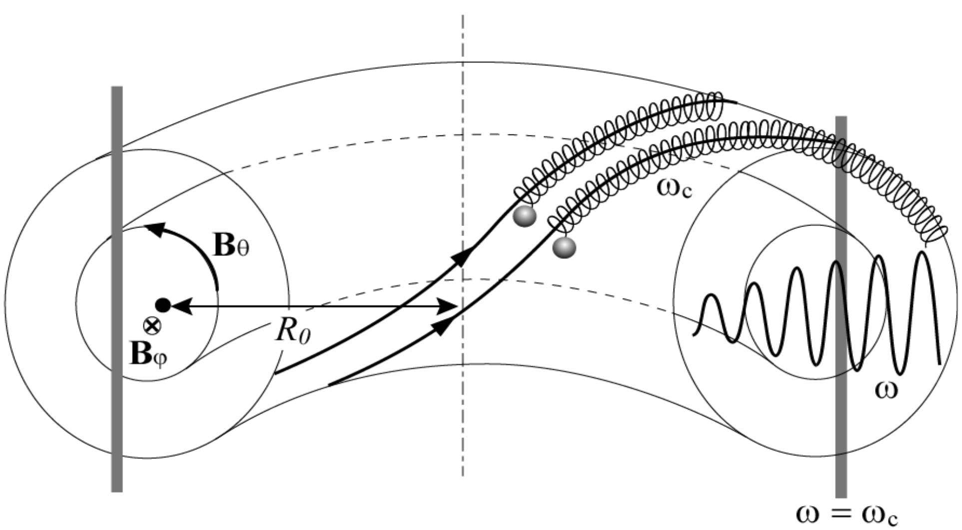
$$2\delta v_c^2(v_c,\delta t)=\mathrm{Re}\left[\mathcal{Z}(\delta t)\mathcal{Z}^*(\delta t)-\mathcal{Z}(0)\mathcal{Z}^*(0)\right]$$

$$\varepsilon\sin^2(x/\varepsilon)/x^2\big|_{\varepsilon\to 0}\to\pi\delta(x)$$

$$\delta v_c=\frac{eE}{2m}\frac{\sin[(\omega_c-\omega)\delta t]}{(\omega_c-\omega)}\to\langle\delta v_c\delta v_c\rangle=\frac{e^2E^2}{4m^2}\frac{\sin^2[(\omega_c-\omega)\delta t]}{(\omega_c-\omega)^2}\to\frac{\pi}{4}\frac{e^2E^2}{m^2}\delta(\omega_c-\omega)\delta t$$

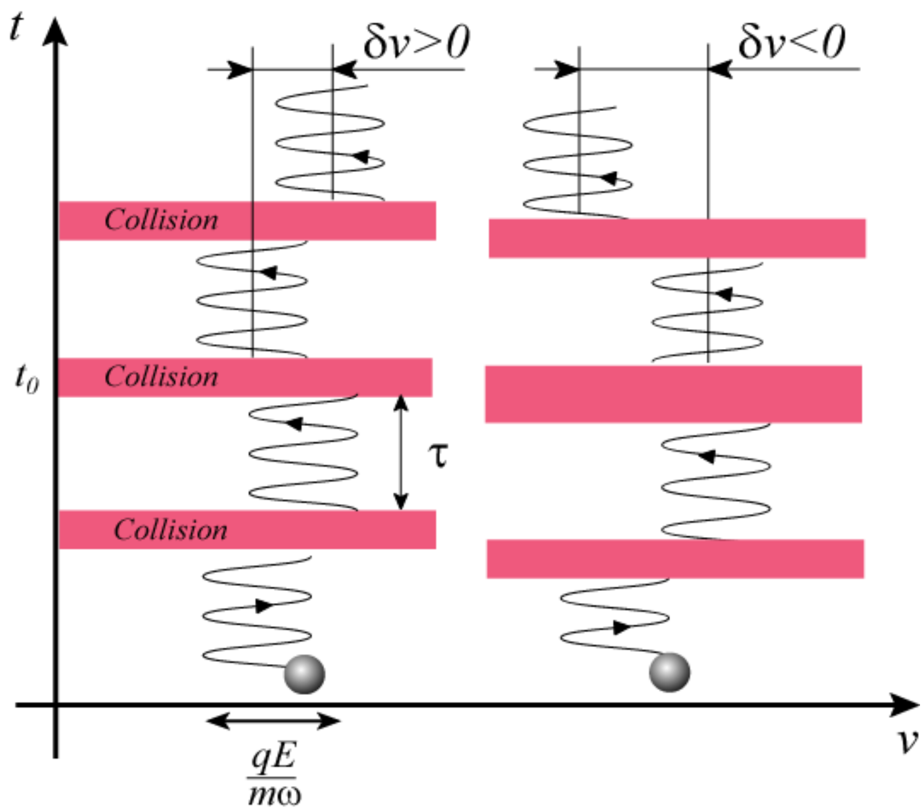
$$w_c=-n\int_0^{+\infty}2\pi m v_c^2\frac{\pi}{4}\frac{e^2E^2}{m^2}\delta(\omega_c-\omega)\frac{\partial f}{\partial v_c}dv_c=-n\frac{\pi}{2}\frac{e^2E^2}{m}\delta(\omega_c-\omega)$$





Inverse Bremsstrahlung

$$\frac{d\mathbf{v}}{dt} = -q\mathbf{E}\sin(\omega t) + \sum_n \delta(t - \tau_n)\,\delta\mathbf{v}_n$$



Emission / Absorption / Emission /Emission / Absorption / Emission...

Bremsstrahlung inverse :

$$\frac{\partial f(v,t)}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v} v^2 \frac{\langle \delta v \delta v \rangle}{2\delta t}\frac{\partial}{\partial v} f(v,t)$$

$$\frac{d\mathbf{v}}{dt} = -q\mathbf{E}\sin(\omega t) + \sum_n \delta(t - \tau_n)\,\delta\mathbf{v}_n$$

$$\mathbf{v}(t) = \mathbf{v}(t_0) + q\mathbf{E}\left[\cos(\omega t) - \cos(\omega t_0)\right]/m\omega$$

$$\cos a - \cos b = -2\sin(a + b/2)\sin(a - b/2)$$

$$\delta\mathbf{v} = \frac{2q}{m\omega}\mathbf{E}\sin\left(\omega\frac{\tau}{2}\right)\sin\left(\omega\frac{\tau}{2} + t_0\right)$$

$$\frac{\langle \delta \mathbf{v} \cdot \delta \mathbf{v} \rangle_{\tau, t_0}}{\delta t} = \frac{2q^2 E^2}{m^2 \omega^2} \frac{\int_0^{+\infty} \exp(-\nu\tau) \left[\sin\left(\omega \frac{\tau}{2}\right)\right]^2 d\tau}{\int_0^{+\infty} \tau \exp(-\nu\tau)\, d\tau}$$

$$\frac{\langle \delta v \delta v \rangle}{\delta t} = \frac{q^2 E^2}{3m^2} \frac{\nu(v)}{\nu^2(v) + \omega^2}$$

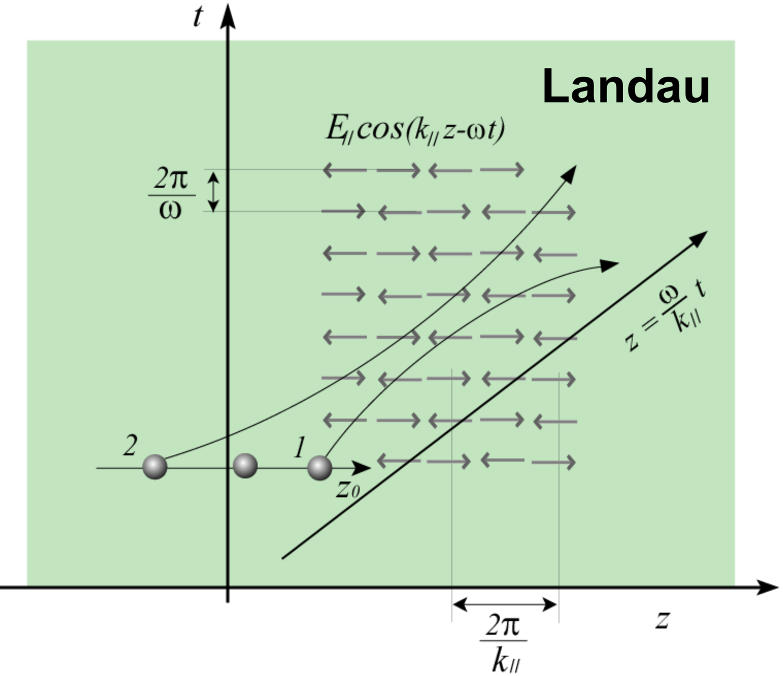
$$\langle w \rangle_B \left[\frac{\mathrm{W}}{\mathrm{m}^3}\right] = n \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \frac{m}{2} v^2 f(v,t)\, 4\pi v^2 dv$$

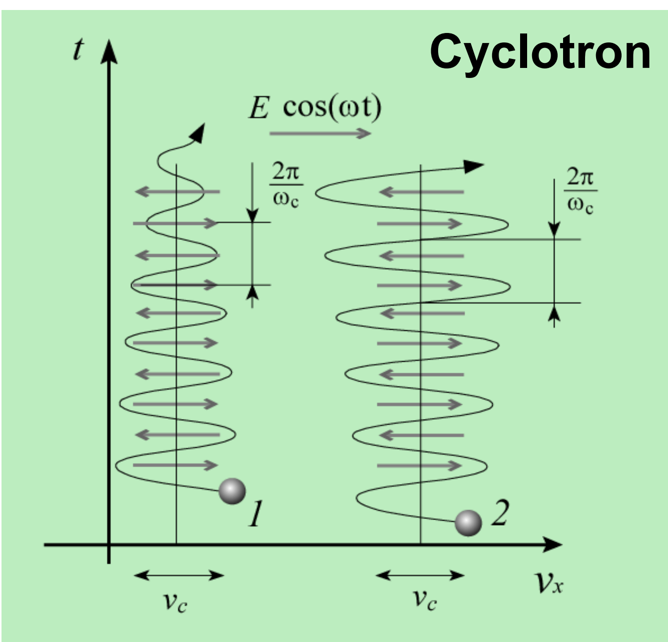
$$\frac{\partial f(v,t)}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} v^2 \frac{\langle \delta v \delta v \rangle}{2\delta t} \frac{\partial}{\partial v} f(v,t)$$

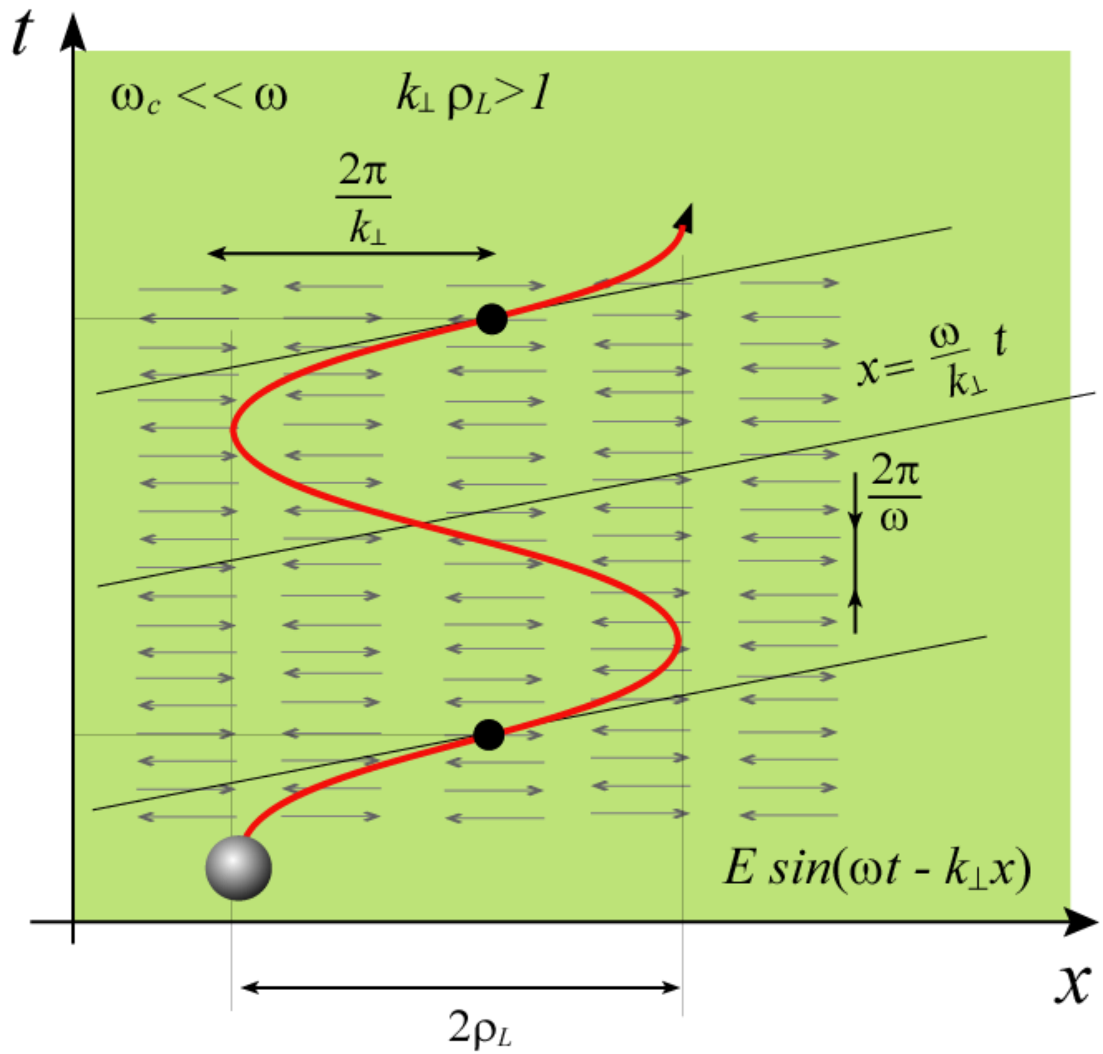
$$\langle w \rangle_B = -nm \int_{-\infty}^{+\infty} v \frac{\langle \delta v \delta v \rangle}{2\delta t} \frac{\partial f}{\partial v} 4\pi v^2 dv$$

$$\langle w \rangle_B = -nE^2 \frac{q^2}{6m} \int_{-\infty}^{+\infty} \frac{\nu(v)}{\nu^2(v) + \omega^2} v \frac{\partial f}{\partial v} 4\pi v^2 dv$$

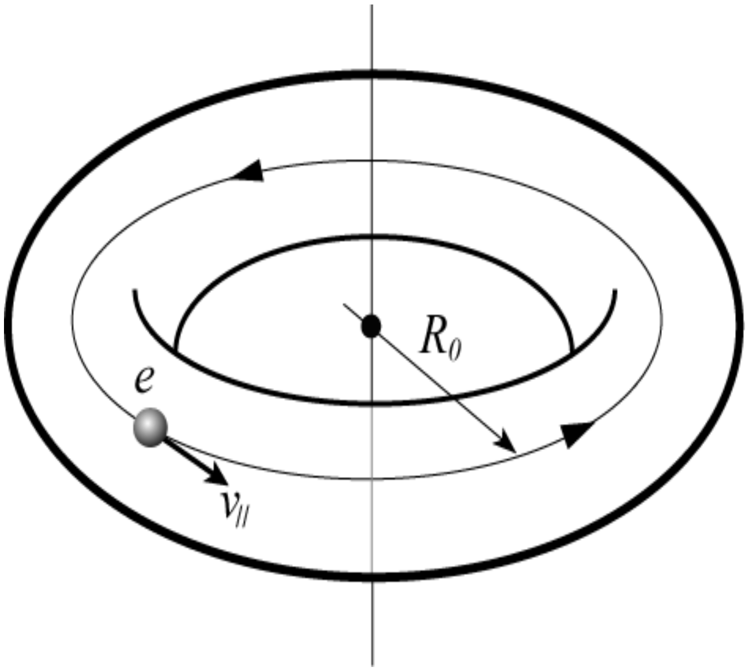
Finite Larmor radius effect : perpendicular Landau absorption







Current Generation



$$Courant\ Toroïdal: \ I = \frac{e v_{\parallel}}{2\pi R_0}$$

$$\mu_{\parallel} = \left| \frac{e}{m_e \nu_e} \right|$$

$$\mu_{\parallel} = \left| \frac{e}{m_e \nu_e} \right| \rightarrow J_{\parallel} = \underbrace{-e^2 n_e \mu_{\parallel} \boldsymbol{\nabla}_{\parallel} \phi}_{\text{Capacitif}} + \underbrace{e^2 n_e \mu_{\parallel} \frac{\partial A_{\parallel}}{\partial t}}_{\text{Inductif}} + \underbrace{e \mu_{\parallel} k_B \boldsymbol{\nabla}_{\parallel} n_e T_e}_{\text{Thermoélectrique}} - \underbrace{e n_e \mu_{\parallel} m_e \frac{\delta v_{\parallel}}{\delta t}}_{\text{Non-inductif}}$$

$$\oint_{2\pi R_0} \boldsymbol{\nabla}_{\parallel} \phi ds = 0$$

$$\oint_{2\pi R_0} \boldsymbol{\nabla}_{\parallel} n_e T_e ds = 0$$

Thermoélectric effect

Spitzer Conductivity

$$e\mu_{\|} k_B \nabla_{\|} n_e T_e$$

$$\mu_{\|} = \left| \frac{e}{m_e \nu_e} \right|$$

$$J_T = \frac{6\sqrt{2}\pi^{\frac{3}{2}} \varepsilon_0^2 k_B^{\frac{5}{2}}}{m_e^{\frac{1}{2}} Z e^3 \Lambda} \frac{T_e^{\frac{3}{2}}}{n_e} \nabla_{\|} n_e T_e$$

$$\frac{T_e^{\frac{3}{2}}}{n_e} \nabla_{\|} n_e T_e = T_e^{\frac{5}{2}} \nabla_{\|} \ln n_e + T_e^{\frac{3}{2}} \nabla_{\|} T_e$$

$$n_e = n_0 \left[1 + \delta \cos\left(\frac{s}{R_0} \right) \right] \;,\; T_e = T_0 \left[1 + \varepsilon \cos\left(\frac{s}{R_0} + \varphi \right) \right]$$

$$J_T = \frac{6\sqrt{2}\pi^{\frac{3}{2}}\varepsilon_0^2 k_B^{\frac{5}{2}}}{m_e^{\frac{1}{2}} Z e^3 \Lambda} \frac{T_e^{\frac{3}{2}}}{n_e} \nabla_{\parallel} n_e T_e$$

$$\frac{T_e^{\frac{3}{2}}}{n_e} \nabla_{\parallel} n_e T_e = T_e^{\frac{5}{2}} \nabla_{\parallel} \ln n_e + T_e^{\frac{3}{2}} \nabla_{\parallel} T_e$$

$$n_e = n_0 \left[1 + \delta \cos\left(\frac{s}{R_0}\right)\right] \qquad T_e = T_0 \left[1 + \varepsilon \cos\left(\frac{s}{R_0} + \varphi\right)\right]$$

$$\downarrow$$

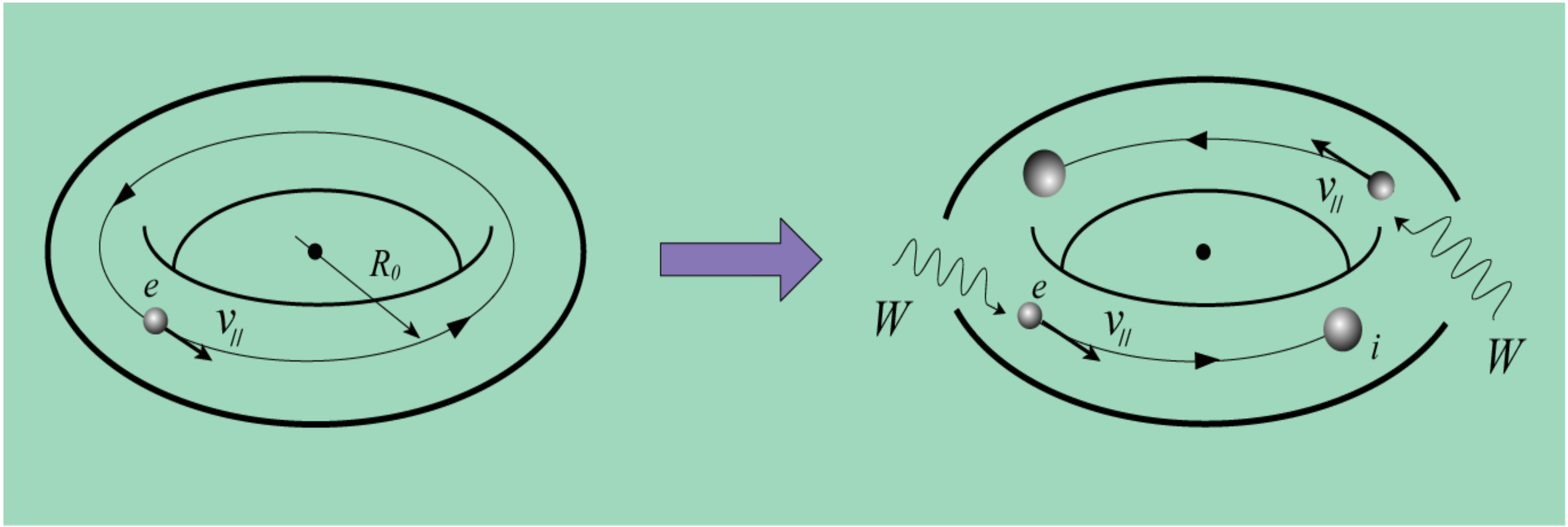
$$\frac{\oint_{2\pi R_0} T_e^{\frac{5}{2}} \nabla_{\parallel} \ln n_e ds}{\oint_{2\pi R_0} ds} = \frac{5T_0^{\frac{5}{2}}}{4R_0} \delta\varepsilon \sin\varphi$$

$$n_e = n_0 \left[1 + \delta \cos\left(\frac{s}{R_0} \right) \right]$$

$$T_e = T_0 \left[1 + \varepsilon \cos\left(\frac{s}{R_0} + \varphi \right) \right]$$

$$\textit{Génération thermoélectrique:} \quad I_T = \frac{15\pi^{\frac{1}{2}} \varepsilon_0^2 m_e^2 v_T^5}{16 Z e^3 \Lambda R_0^2 n_e} \delta \varepsilon \sin\varphi$$

Current generation I : 1D response



$$\textit{Toroïdal current}: \; I\,[\mathrm{A}] \equiv \frac{e\,[\mathrm{C}]\, v_{\parallel}\,[\mathrm{m/s}]}{2\pi R_0\,[\mathrm{m}]} \rightarrow I = \frac{e\left\langle v_{\parallel}\right\rangle}{2\pi R_0}$$

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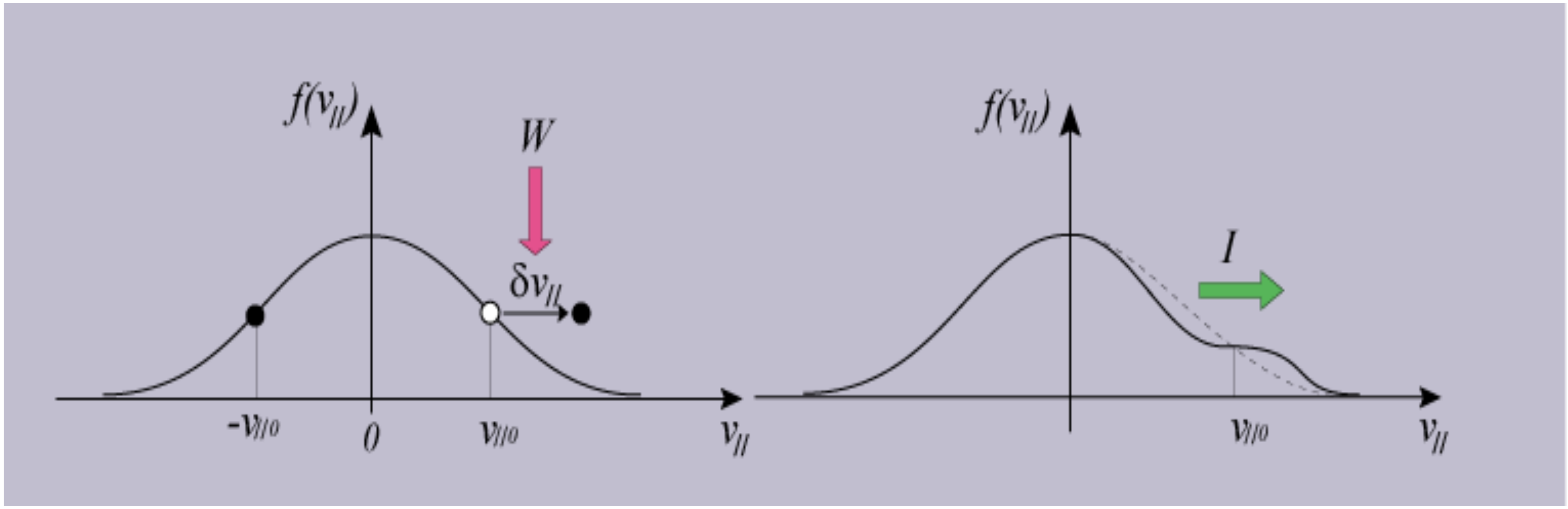
$$v_{\parallel 0} = \frac{\omega}{k_\parallel} \rightarrow W = -\pi\omega\frac{\omega_p^2}{k_\parallel^2}\left.\frac{\partial f}{\partial v_\parallel}\right|_{v_{\parallel 0}=\frac{\omega}{k}} \frac{\varepsilon_0 E^2}{2}$$



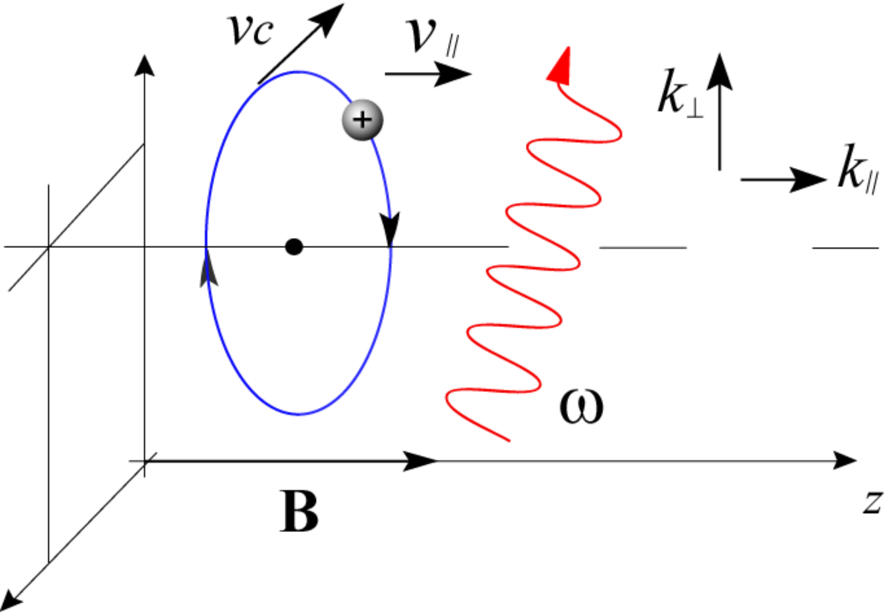
$$\frac{dv_\parallel}{dt} = -\nu\left(v_\parallel - v_{\parallel 0}\right) + \delta v_\parallel \delta(t) \rightarrow v_\parallel(t) = v_{\parallel 0} + \theta(t)\,\delta v_\parallel \exp(-\nu t)$$

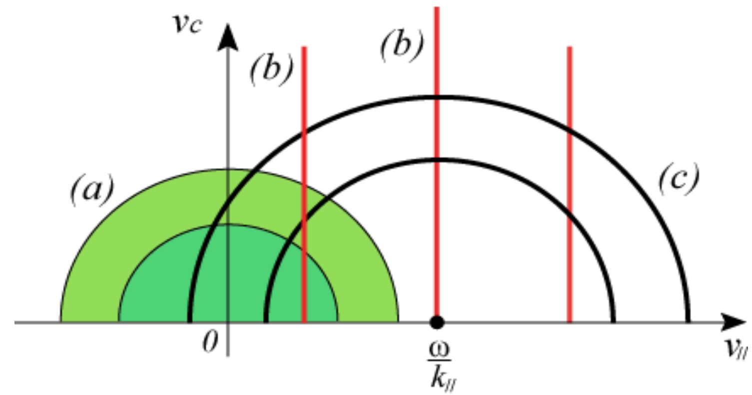
$$v_\parallel(t) = v_{\parallel 0} + \theta(t)\,\delta v_\parallel \exp(-\nu t) \rightarrow \left\langle v_\parallel(t)\right\rangle = \theta(t)\,\delta v_\parallel \exp(-\nu t)$$

$$\textit{Fisch efficiency}: \frac{I}{W}\left[\frac{\mathrm{A}}{\mathrm{W}}\right] \equiv \frac{\int \mathcal{I}(t)\,dt}{\int \mathcal{W}(t)\,dt} = \frac{e}{2\pi R_0 m_e v_{\parallel 0}\nu}$$

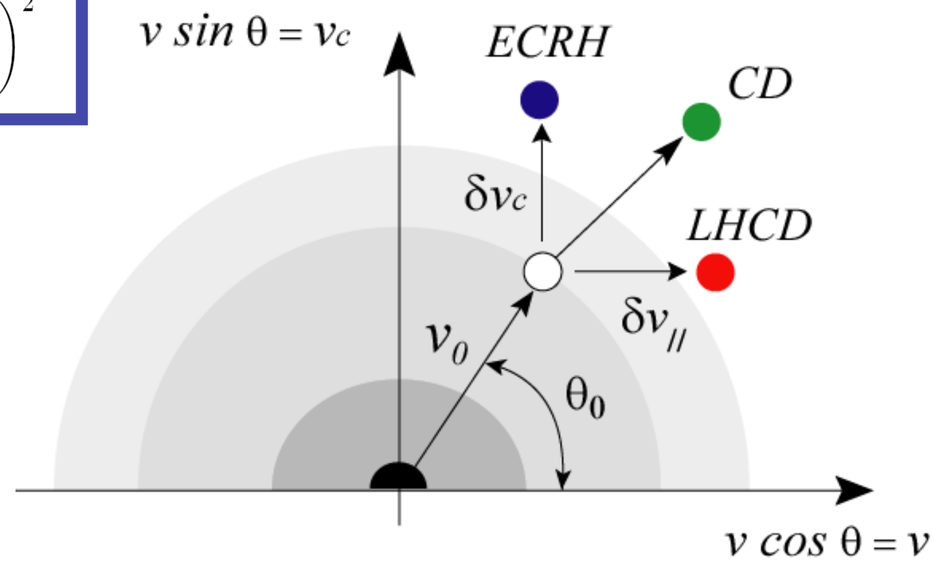
$$\mathcal{W}(t)\,[\mathrm{W}] = m_e v_{\parallel 0}\delta v_\parallel \delta(t)$$

$$\mathcal{I}(t)\,[\mathrm{A}] = \frac{e\left\langle v_\parallel\right\rangle}{2\pi R_0} = \frac{e\theta(t)}{2\pi R_0}\delta v_\parallel \exp -\nu t$$



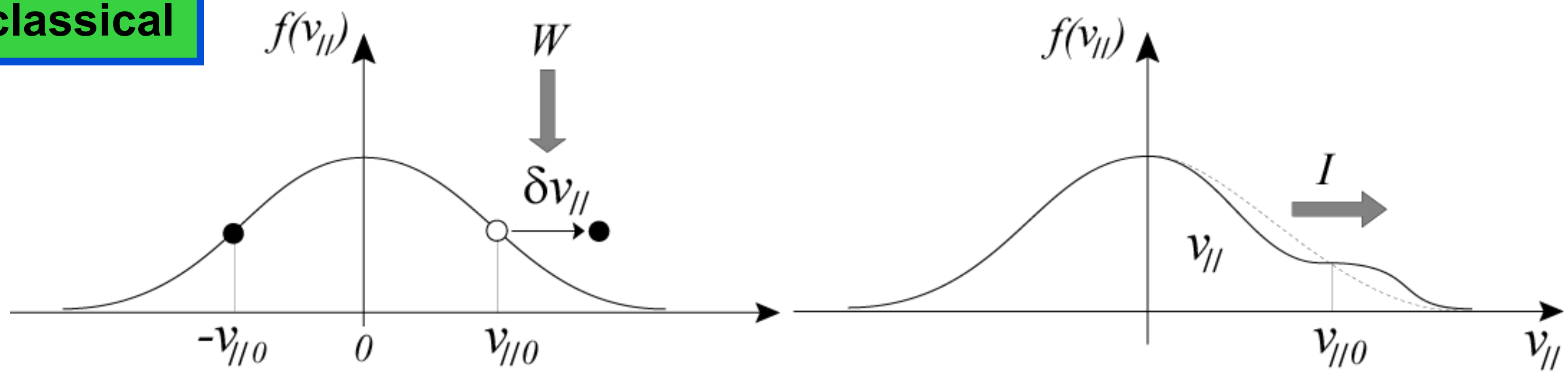


Resonance curves (b)	:	$\omega = N\omega_c + k_\parallel v_\parallel$
Isoenergy lines (a)	:	$H = \frac{m}{2}v_c^2 + \frac{m}{2}v_\parallel^2$
Diffusion path (c)	:	$v_c^2 + \left(v_\parallel - \frac{\omega}{k_\parallel}\right)^2 = v_{0c}^2 + \left(v_{0\parallel} - \frac{\omega}{k_\parallel}\right)^2$

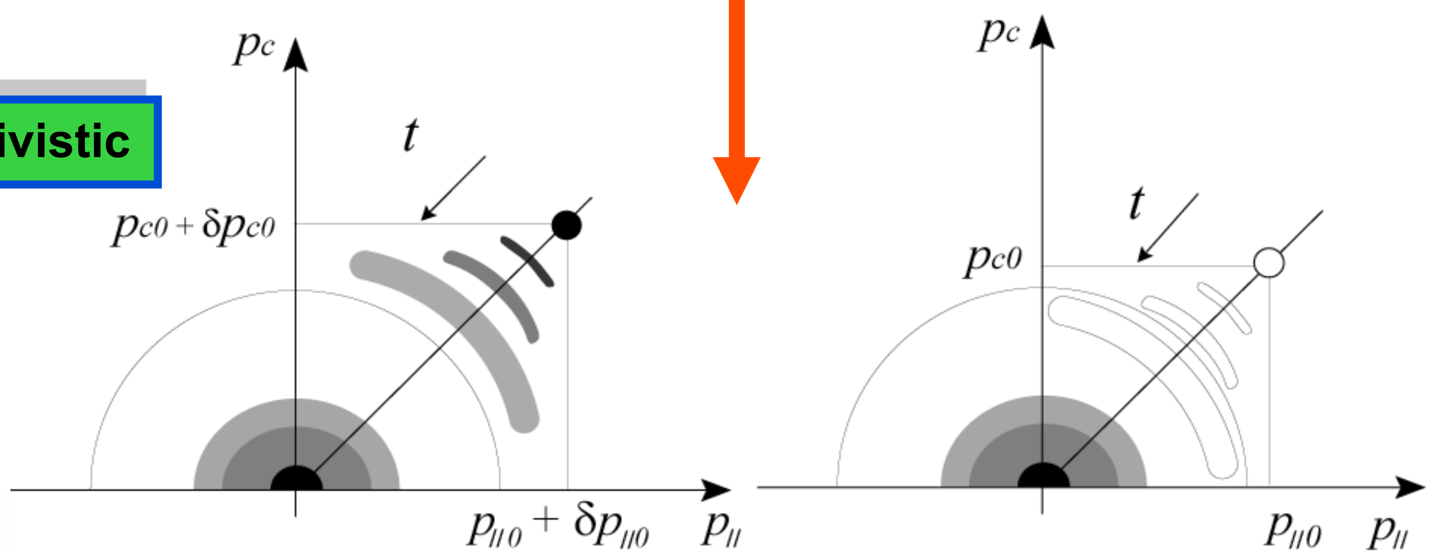


Current generation I : 2D response

1D classical



2D relativistic



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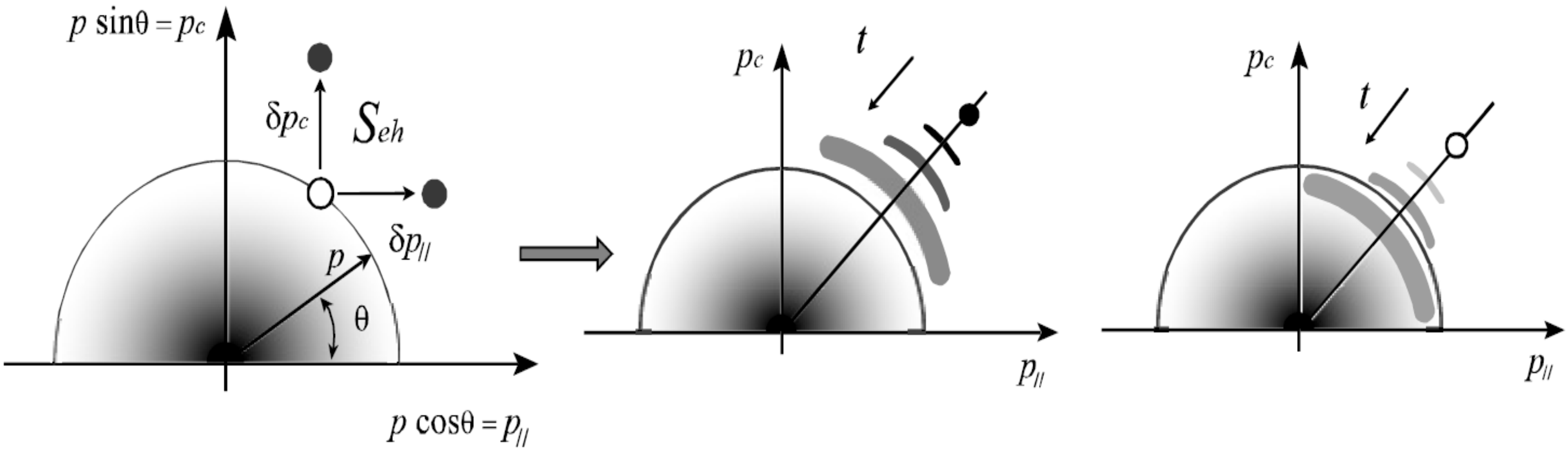
2 D Model

$$m = e = c = \tau_e = 4\pi\varepsilon_0^2 m_e^2 c^3 / e^4 n_e \Lambda = 1$$



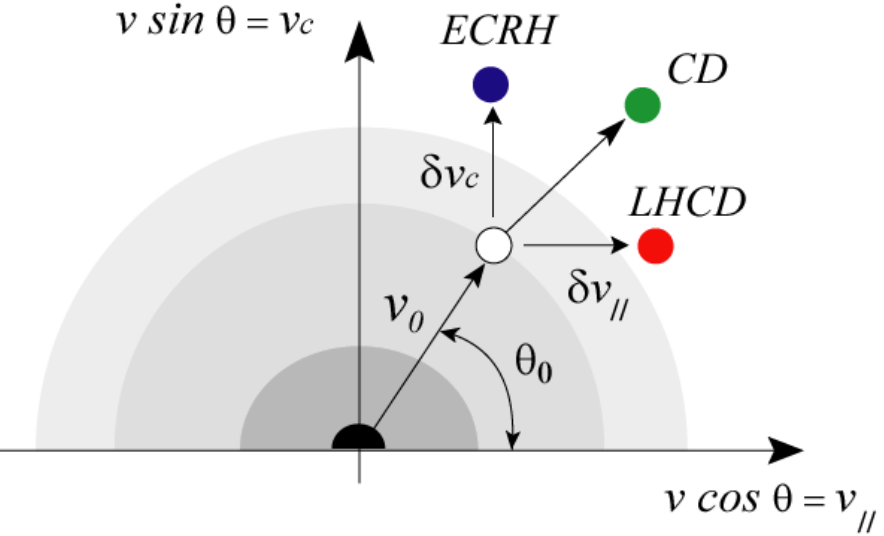
$$\gamma^2 = 1 + p_{\|}^2 + p_c^2$$

$$\gamma\omega - k_{\|}\gamma v_{\|} = n\omega_c \;\longrightarrow$$

$$\delta\gamma = \hbar\omega$$

$$\delta p_{\|} = \hbar k_{\|}$$

$$\delta p_c = \frac{n\hbar\omega_c}{p_c}$$



Photon absorption

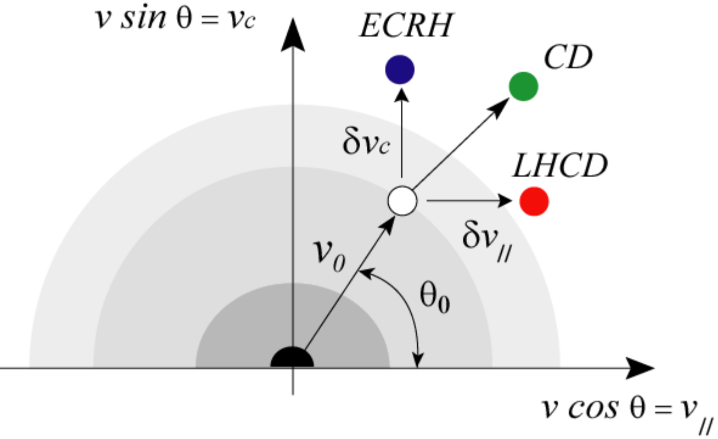
$$\delta \mathbf{p}_0 \cdot \frac{\partial}{\partial \mathbf{p}_0} \delta\left(\mathbf{p} - \mathbf{p}_0\right) = \hbar \left[\frac{n\omega_c}{p_{0c}} \frac{\partial}{\partial p_{0c}} + k_\parallel \frac{\partial}{\partial p_{0\parallel}} \right] \delta\left(\mathbf{p} - \mathbf{p}_0\right)$$

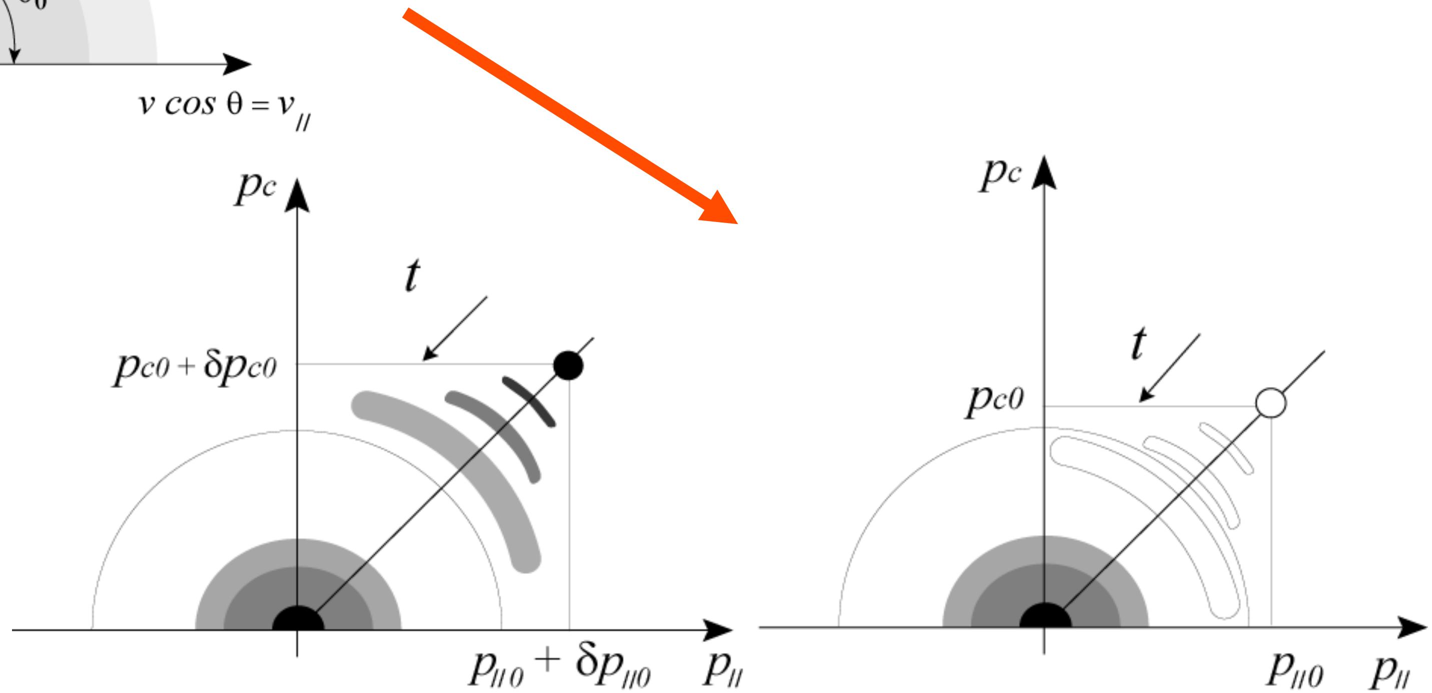
$$\mathrm{L}_n\left(\mathbf{p}\right) \equiv \frac{n\omega_c}{p_c} \frac{\partial}{\partial p_c} + k_\parallel \frac{\partial}{\partial p_\parallel}$$

$$\mathrm{L}_n\left(\mathbf{p}\right) \equiv \frac{\gamma\omega}{p} \frac{\partial}{\partial p} + \frac{\gamma\omega \sin^2\theta - n\omega_c}{p^2 \cos\theta} \frac{\partial}{\partial \cos\theta}$$

$$S\left(\mathbf{p}\right) = \frac{k_\parallel}{\omega} \frac{\partial}{\partial p_\parallel} + \frac{n\omega_c}{\omega} \frac{1}{p_c} \frac{\partial}{\partial p_c} = \frac{\gamma}{p} \frac{\partial}{\partial p} + \frac{\gamma}{p^2 \cos\theta} \left(\sin^2\theta - \frac{n\omega_c}{\gamma\omega} \right) \frac{\partial}{\partial \cos\theta}$$

Electron-hole collisional relaxation





$$\underbrace{\frac{\partial f}{\partial t}} - \underbrace{C_r \cdot f}_{\text{collisions}} = \underbrace{\delta\left(t\right)\frac{\delta\mathcal{U}\left(\mathbf{p}_0\right)}{\omega}\mathrm{L}_n\left(\mathbf{p}_0\right)\cdot\delta\left(\mathbf{p}-\mathbf{p}_0\right)}$$

Evolution **Collisions** **Photons Absorption**

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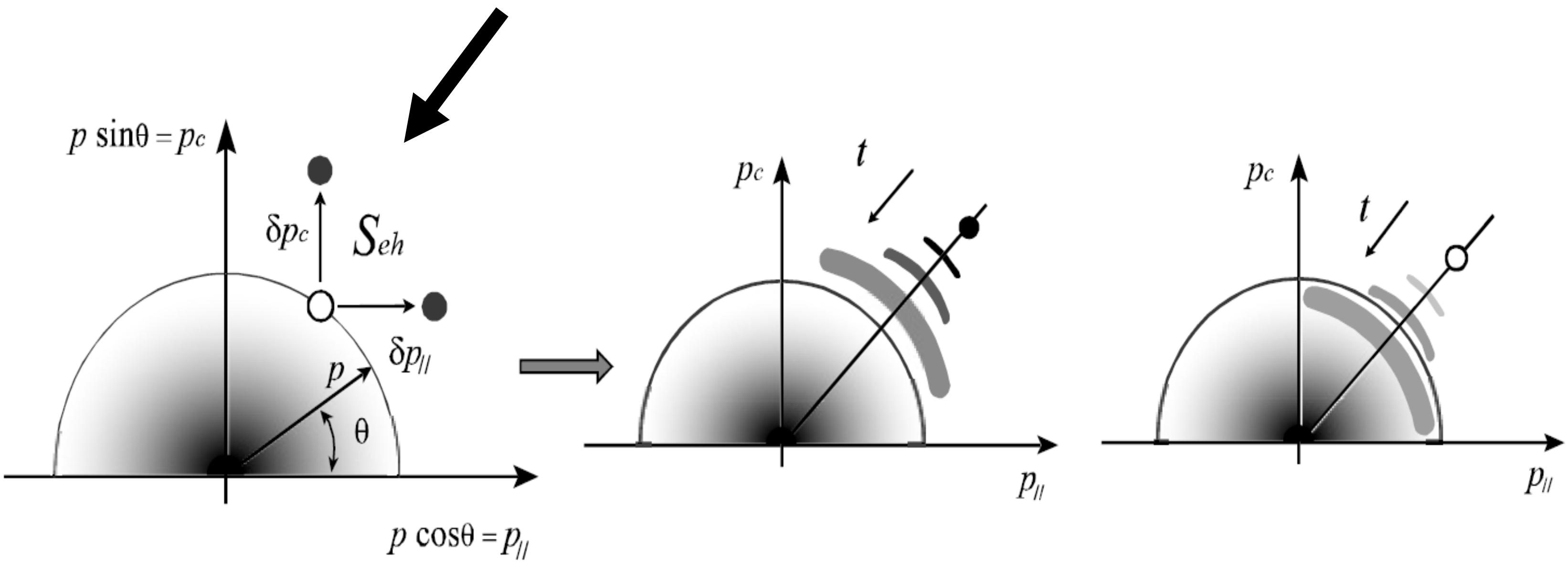
Excitation - Relaxation

$$\delta\left(\mathbf{p}-\mathbf{p}_0-\delta\mathbf{p}_0\right)-\delta\left(\mathbf{p}-\mathbf{p}_0\right)$$



$$C \equiv \frac{1}{p^2}\frac{\partial}{\partial p}\gamma^2 f_r + \frac{Z+1}{2}\frac{\gamma}{p^3}\frac{1}{\sin\theta}\left(\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\right)$$

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Excitation - Relaxation

$$\delta U\left(k_{\|},\omega\right)=N\hbar\omega$$

$$\frac{\partial f}{\partial t}-C\cdot f=\delta U\left(k_{\|},\omega,t_0\right)S\cdot\delta\left(\mathbf{p}-\mathbf{p}_0\right)\delta\left(t-t_0\right)$$

$$S\left(\mathbf{p}\right)=\frac{k_{\|}}{\omega}\frac{\partial}{\partial p_{\|}}+\frac{n\omega_c}{\omega}\frac{1}{p_c}\frac{\partial}{\partial p_c}=\frac{\gamma}{p}\frac{\partial}{\partial p}+\frac{\gamma}{p^2\cos\theta}\left(\sin^2\theta-\frac{n\omega_c}{\gamma\omega}\right)\frac{\partial}{\partial\cos\theta}$$

$$\exp Ct\equiv 1+Ct+\frac{Ct\cdot Ct}{2!}+\frac{Ct\cdot Ct\cdot Ct}{3!}+\dots$$

$$f\left(\mathbf{p},\mathbf{p}_0,t-t_0\right)=\theta\left(t-t_0\right)\exp\left[C\left(\mathbf{p}\right)\left(t-t_0\right)\right]\cdot U\left(k_{\|},\omega\right)S\left(\mathbf{p}_0\right)\cdot\delta\left(\mathbf{p}-\mathbf{p}_0\right)$$

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$$\textit{Toroïdal current}: \ I\,[\mathrm{A}] \equiv \frac{e\,[\mathrm{C}]\,v_\parallel\,[\mathrm{m/s}]}{2\pi R_0\,[\mathrm{m}]} \rightarrow I = \frac{e\left\langle v_\parallel \right\rangle}{2\pi R_0}$$

$$\mathcal{I}\left(\mathbf{p}_0, t - t_0\right) = \frac{1}{2\pi R_0} \int v_\parallel f d\mathbf{p} = \frac{e\theta\left(t - t_0\right)}{2\pi R_0} \delta U\left(k_\parallel, \omega\right) S\left(\mathbf{p}_0\right) \cdot \int v_\parallel \exp\left[C\left(\mathbf{p}\right) t\right] \cdot \delta\left(\mathbf{p} - \mathbf{p}_0\right) d\mathbf{p}$$

$$\delta U\left(k_\parallel, \omega\right) = W\left(k_\parallel, \omega\right) \delta t$$

$$I\left(\mathbf{p}_0\right) = \int_0^{+\infty} \mathcal{I}\left(\mathbf{p}_0, t\right)$$

$$\frac{I}{W}\left[\frac{\mathrm{A}}{\mathrm{W}}\right] = -\frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda \omega R_0} \mathrm{L}_n\left(\mathbf{p}_0\right) \cdot \int d\mathbf{p} v_\parallel\left(\mathbf{p}\right) C_r^{-1}\left(\mathbf{p}\right) \cdot \delta\left(\mathbf{p} - \mathbf{p}_0\right)$$

$$\mathrm{L}_n\left(\mathbf{p}\right) \equiv \frac{n\omega_c}{p_c} \frac{\partial}{\partial p_c} + k_\parallel \frac{\partial}{\partial p_\parallel}$$

$$\mathrm{L}_n\left(\mathbf{p}\right) \equiv \frac{\gamma\omega}{p} \frac{\partial}{\partial p} + \frac{\gamma\omega \sin^2\theta - n\omega_c}{p^2 \cos\theta} \frac{\partial}{\partial \cos\theta}$$

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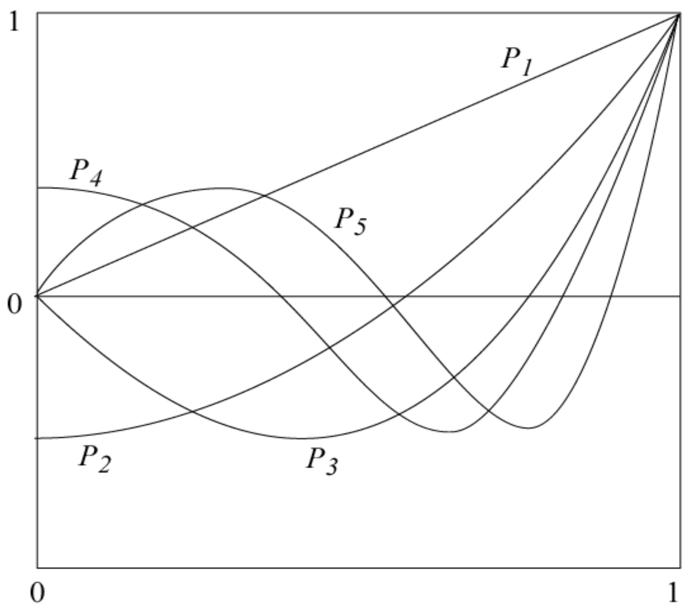
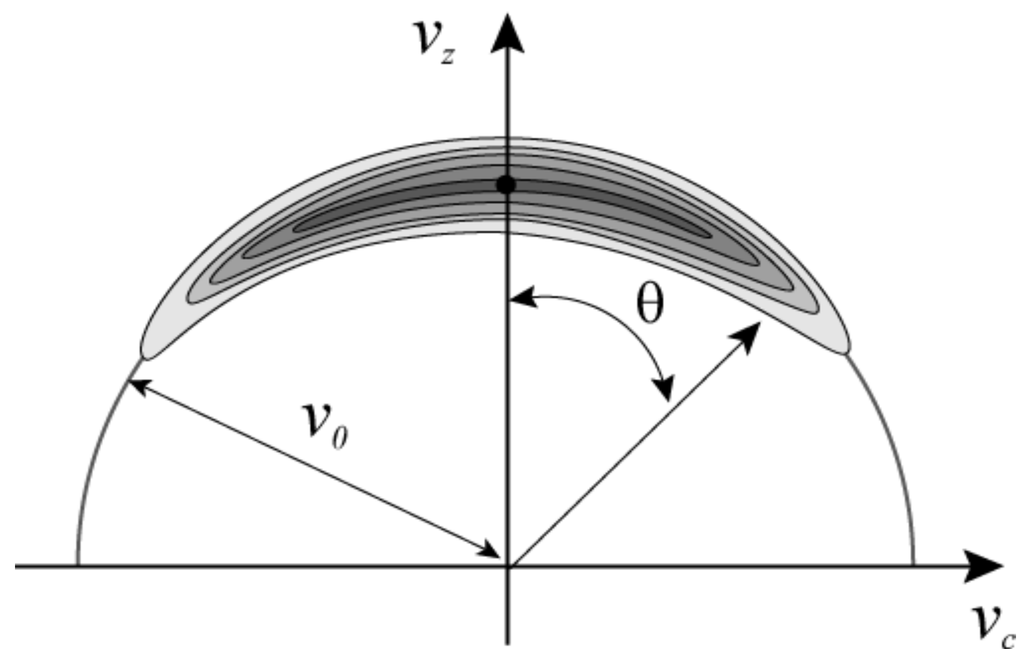
$$S(\mathbf{p}) = \frac{k_{\parallel}}{\omega}\frac{\partial}{\partial p_{\parallel}} + \frac{n\omega_c}{\omega}\frac{1}{p_c}\frac{\partial}{\partial p_c} = \frac{\gamma}{p}\frac{\partial}{\partial p} + \frac{\gamma}{p^2\cos\theta}\left(\sin^2\theta - \frac{n\omega_c}{\gamma\omega}\right)\frac{\partial}{\partial\cos\theta}$$

$$C^{-1}\cdot\delta(\mathbf{p}-\mathbf{p}_0) = -\theta(p_0 - p)\sum_{l=0}^{l=+\infty}\frac{(2l+1)}{4\pi\gamma^2}\left[\frac{p(\gamma_0+1)}{p_0(\gamma+1)}\right]^{\frac{(Z+1)l(l+1)}{2}} P_l(\cos\theta)\,P_l(\cos\theta_0)$$

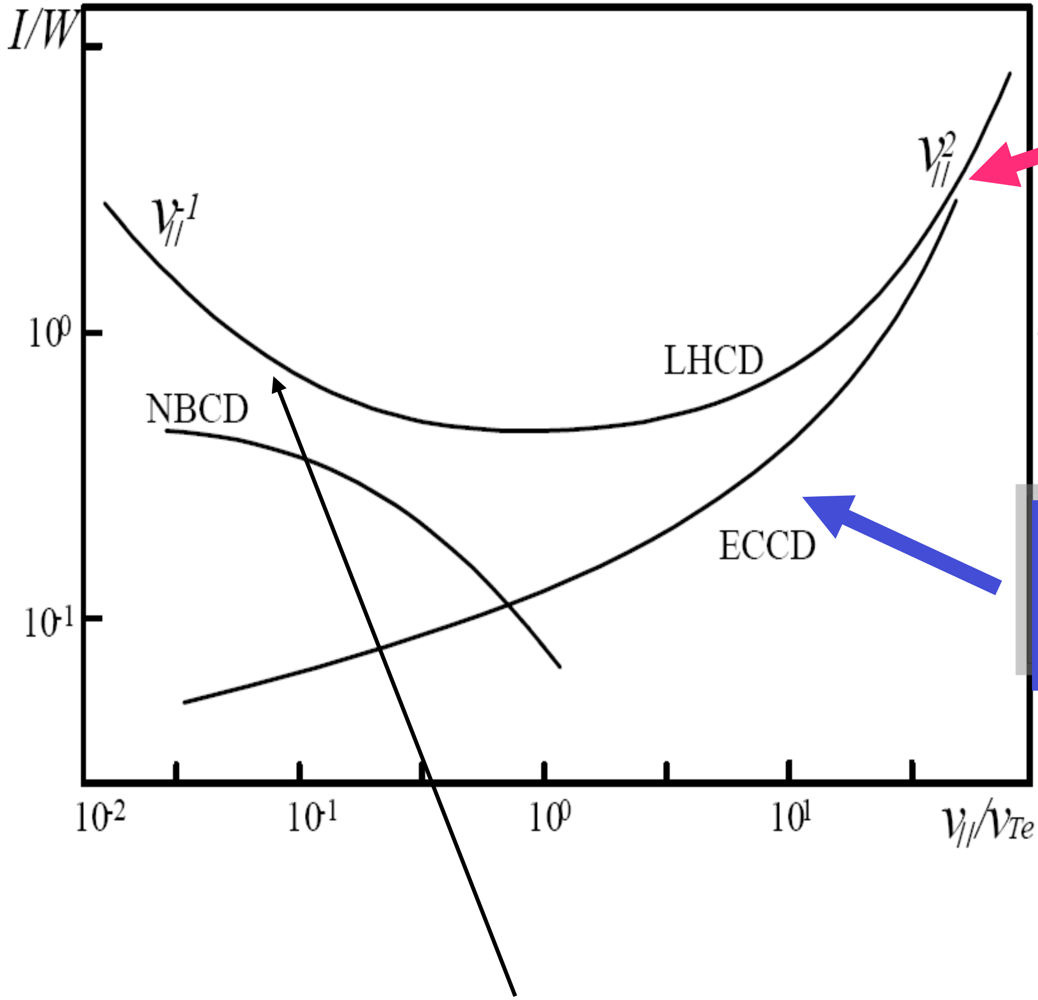




$$C^{-1}\cdot\delta\left(\mathbf{p}-\mathbf{p}_0\right)=-\theta\left(p_0-p\right)\sum_{l=0}^{l=+\infty}\frac{(2l+1)}{4\pi\gamma^2}\left[\frac{p\left(\gamma_0+1\right)}{p_0\left(\gamma+1\right)}\right]^{\frac{(Z+1)l(l+1)}{2}}P_l\left(\cos\theta\right)P_l\left(\cos\theta_0\right)$$

$$S\left(\mathbf{p}_0\right)\;=\;\left[\frac{k_\parallel}{\omega}\frac{\partial}{\partial p_{0\parallel}}+\frac{n\omega_c}{\omega}\frac{1}{p_{0c}}\frac{\partial}{\partial p_{0c}}\right]$$

$$\boxed{\frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda R_0}\approx 0.3\left[\frac{\mathrm{A}}{\mathrm{W}}\right]\times\left[\frac{10^{20}\mathrm{m}^{-3}}{n_e}\right]}$$



$$\left.\frac{I}{W}\right|_{\parallel}\left[\frac{\mathrm{A}}{\mathrm{W}}\right]=\frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda R_0}\,\frac{4}{(Z+5)\,N_\parallel^2}$$

$$\left.\frac{I}{W}\right|_{\perp}\left[\frac{\mathrm{A}}{\mathrm{W}}\right]=\frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda R_0}\,\frac{3}{(Z+5)\,N_\parallel^2}$$

$$\frac{I}{W}\left[\frac{\mathrm{A}}{\mathrm{W}}\right]=\frac{3\pi^{\frac{1}{2}}\varepsilon_0^2 m_e}{2R_0 e^3 Z\Lambda n_e}\,\frac{v_T^3}{v_\parallel}$$

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RF Current Transport

$$m = e = c = \tau_e = 4\pi\varepsilon_0^2 m_e^2 c^3 / e^4 n_e \Lambda = 1$$

$$\underbrace{-\frac{1}{p^2}\frac{\partial G}{\partial p} - \frac{Z+1}{2p^3}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial G}{\partial\theta}}_{Collisions} - \underbrace{\frac{D(p)}{r}\frac{\partial}{\partial r} r \frac{\partial G}{\partial r}}_{Transport\ radial} =$$

$$\underbrace{\frac{\delta(p-p_0)}{p^2}\frac{\delta(r-r_0)}{r}\delta(\cos\theta - \cos\theta_0)}_{Excitation}$$

$$G(\mathbf{p}, r, \mathbf{p}_0, r_0) = \theta(p_0 - p)\sum_{l=0}^{l=+\infty}\sum_{J_0(k)=0}(2l+1)\exp\left(-k^2\int_p^{p_0} u^2 D(u)\,du\right)$$

$$\times\left[\frac{p}{p_0}\right]^{\frac{(Z+1)l(l+1)}{2}}\frac{J_0(kr)}{J_1(k)}\frac{J_0(kr_0)}{J_1(k)}P_l(\cos\theta)P_l(\cos\theta_0)$$

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$$\frac{I(r)}{W(r_0)} = \eta(r, r_0) = \frac{1}{2\pi R_0}\left[\frac{1}{p_0}\frac{\partial}{\partial p_0} + \frac{\sin^2\theta_0}{p_0^2\cos\theta_0}\frac{\partial}{\partial\cos\theta_0}\right]\int p\cos\theta G(\mathbf{p}, r, \mathbf{p}_0, r_0)\, d\mathbf{p}$$

$$\eta(r, r_0) = \frac{1}{2\pi R_0}\sum_{J_0(k)=0}\frac{J_0(kr)}{J_1(k)}\frac{J_0(kr_0)}{J_1(k)}\frac{1}{p_0}\frac{\partial}{\partial p_0}\int_0^{p_0}\frac{p^{Z+4}}{p_0^{Z+1}}\exp\left(-k^2\int_p^{p_0}u^2 D(u)\, du\right) dp$$

$$D = \frac{a^2}{\tau_d}$$

$$\left\langle\frac{I}{W}\right\rangle = \left.\frac{I}{W}\right|_{\parallel}\frac{N_{\parallel}^3(Z+5)}{2k^3 J_1(k)}\frac{\tau_d}{\tau_e}$$

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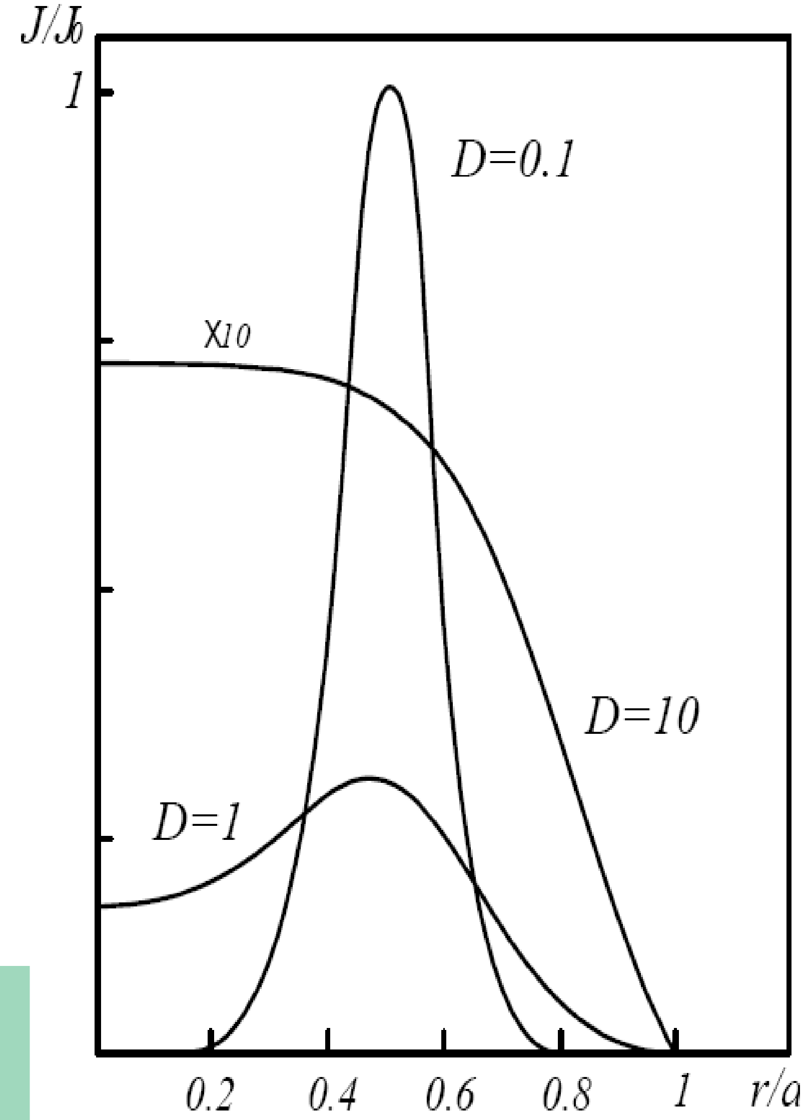
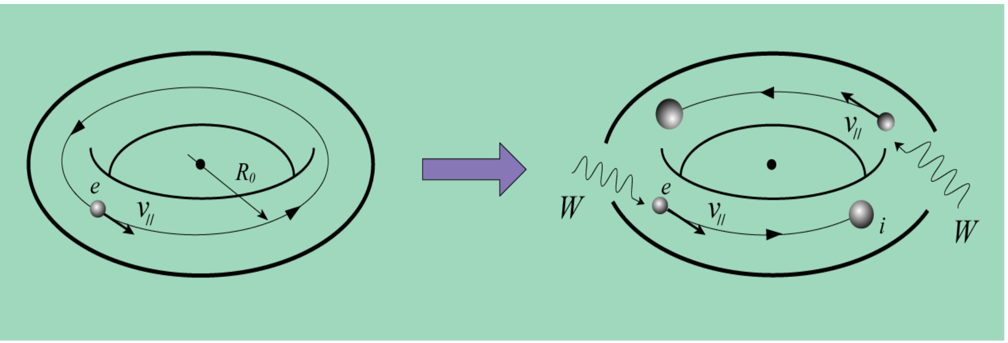
$$\left\langle \frac{I}{W} \right\rangle = \frac{I}{W}\bigg|_{\parallel} \frac{N_{\parallel}^{3}(Z+5)}{2k^{3}J_{1}(k)} \frac{\tau_d}{\tau_e}$$

$$\Gamma = \sqrt{\frac{7}{2(Z+8)N_{\parallel}^{3}} \frac{\tau_e}{\tau_d} a}$$





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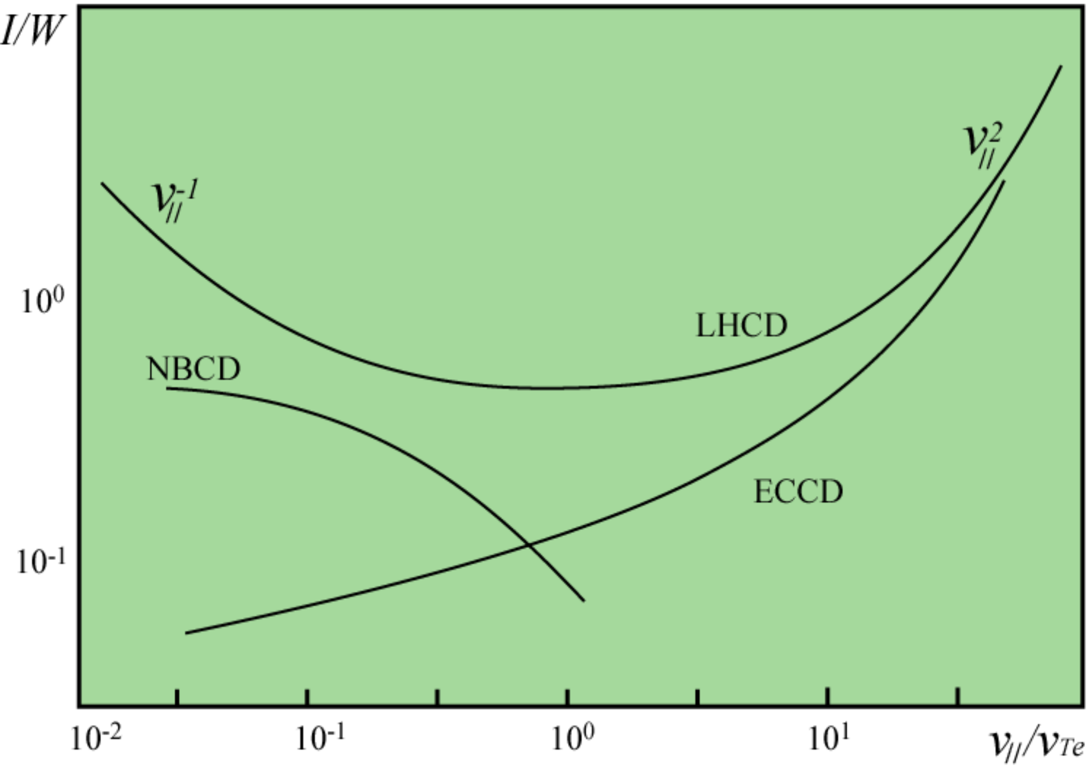

$$\left\langle \frac{I}{W} \right\rangle = \left.\frac{I}{W}\right|_{\|} \frac{N_{\|}^3 (Z+5)}{2k^3 J_1(k)} \frac{\tau_d}{\tau_e}$$

Landau : $$\left.\frac{I}{W}\right|_{\|} \left[\frac{\mathrm{A}}{\mathrm{W}}\right] = \frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda R_0} \frac{4N_{\|}^{-2}}{(Z+5)}$$

Cyclotron : $$\left.\frac{I}{W}\right|_{\perp} \left[\frac{\mathrm{A}}{\mathrm{W}}\right] = \frac{2\varepsilon_0^2 m_e c^2}{e^3 n_e \Lambda R_0} \frac{3N_{\|}^{-2}}{(Z+5)}$$

$$\Gamma = \sqrt{\frac{7}{2(Z+8) N_{\|}^3} \frac{\tau_e}{\tau_d} a}$$

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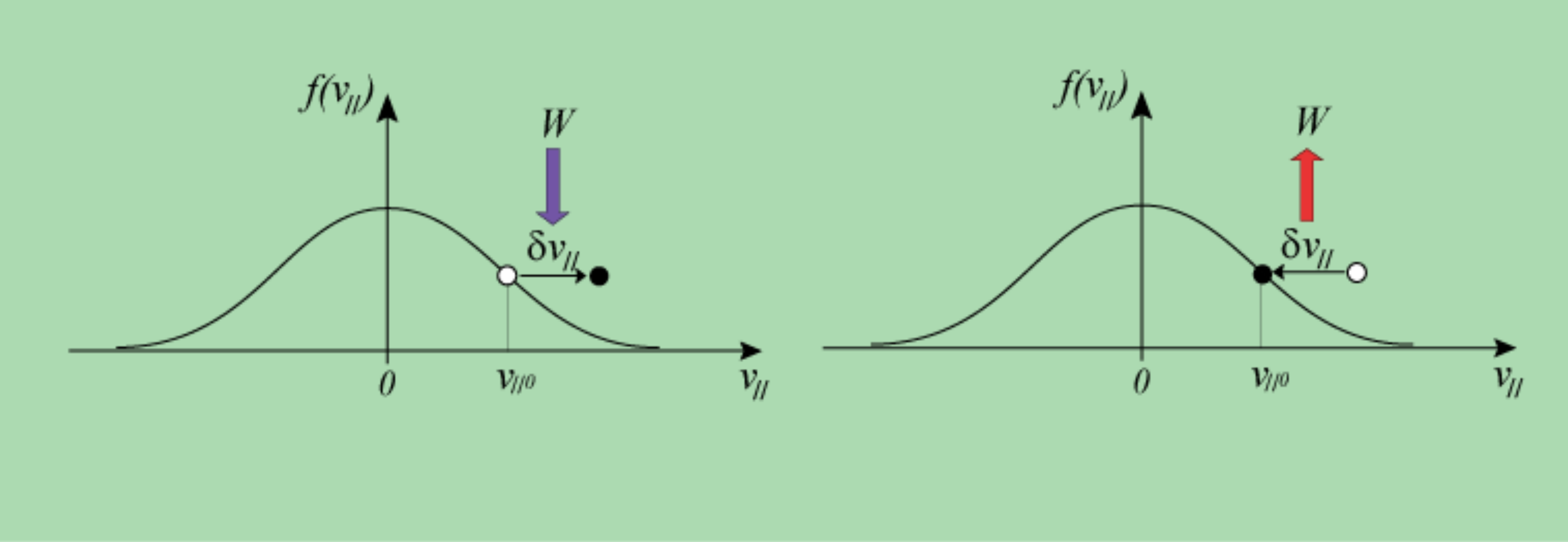
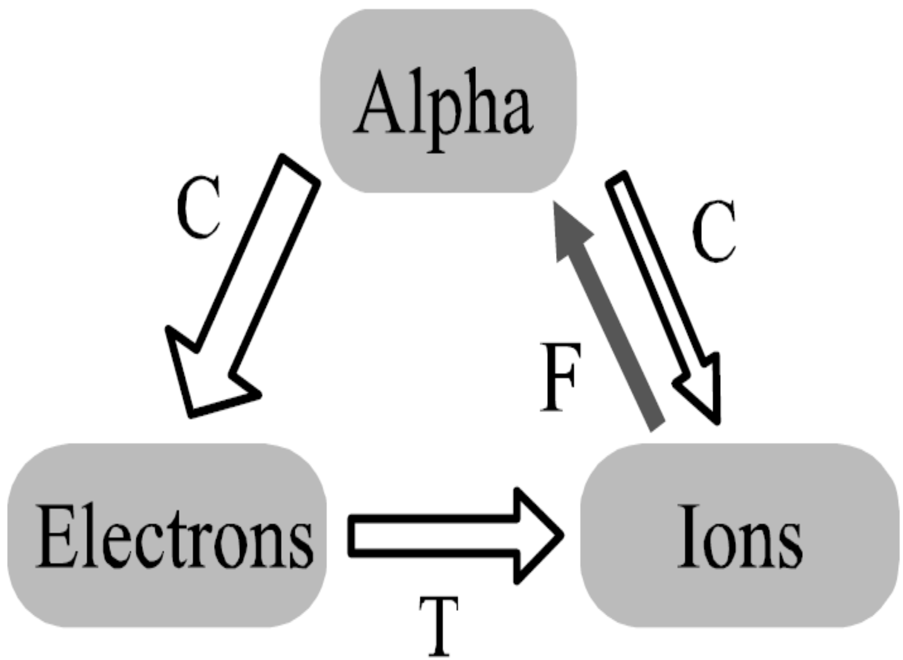
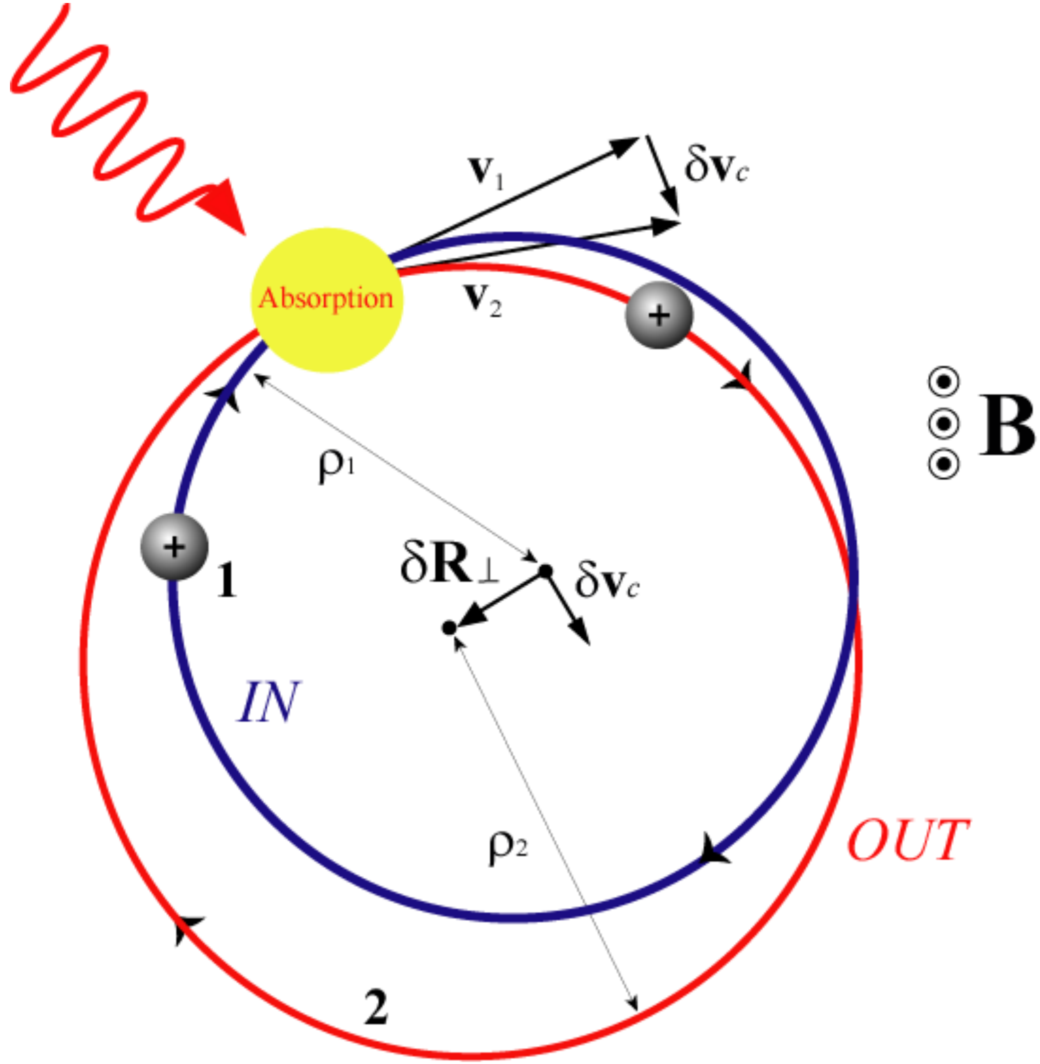
Free Energy Extraction

Emission / Absorption / Emission /Emission / Absorption / Emission...




Alpha
C
C
F
Electrons
Ions
T



$$\delta \mathbf{R}_{\perp} = \frac{\delta \mathbf{v}_c \times \mathbf{b}}{\omega_c}$$

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Perpendicular Diffusion

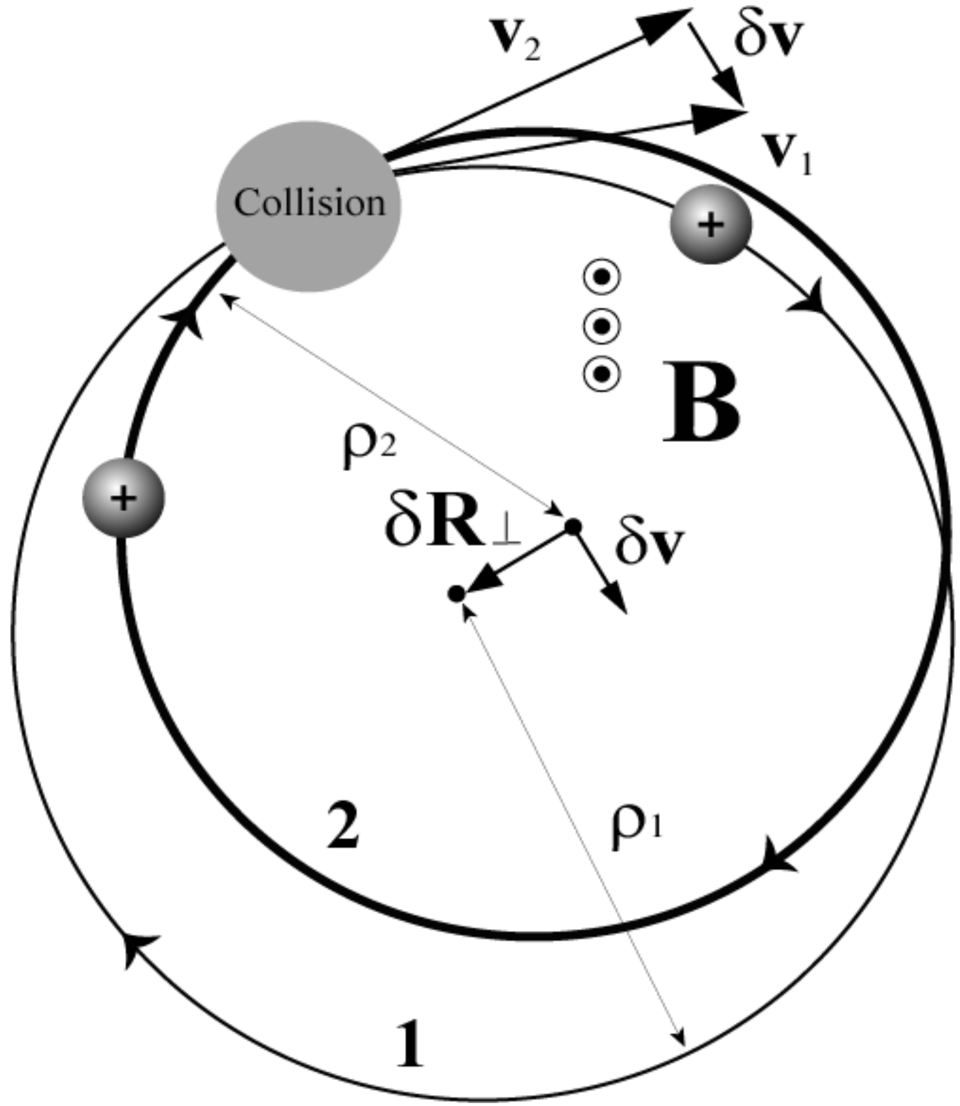
$$\mathbf{r} = \mathbf{R}_\perp + \boldsymbol{\rho}_L$$

$$\omega_c \boldsymbol{\rho}_L = \mathbf{b} \times \mathbf{v}_c$$

$$\mathbf{R}_\perp = \mathbf{r} - \mathbf{b} \times \mathbf{v}_c / \omega_c$$

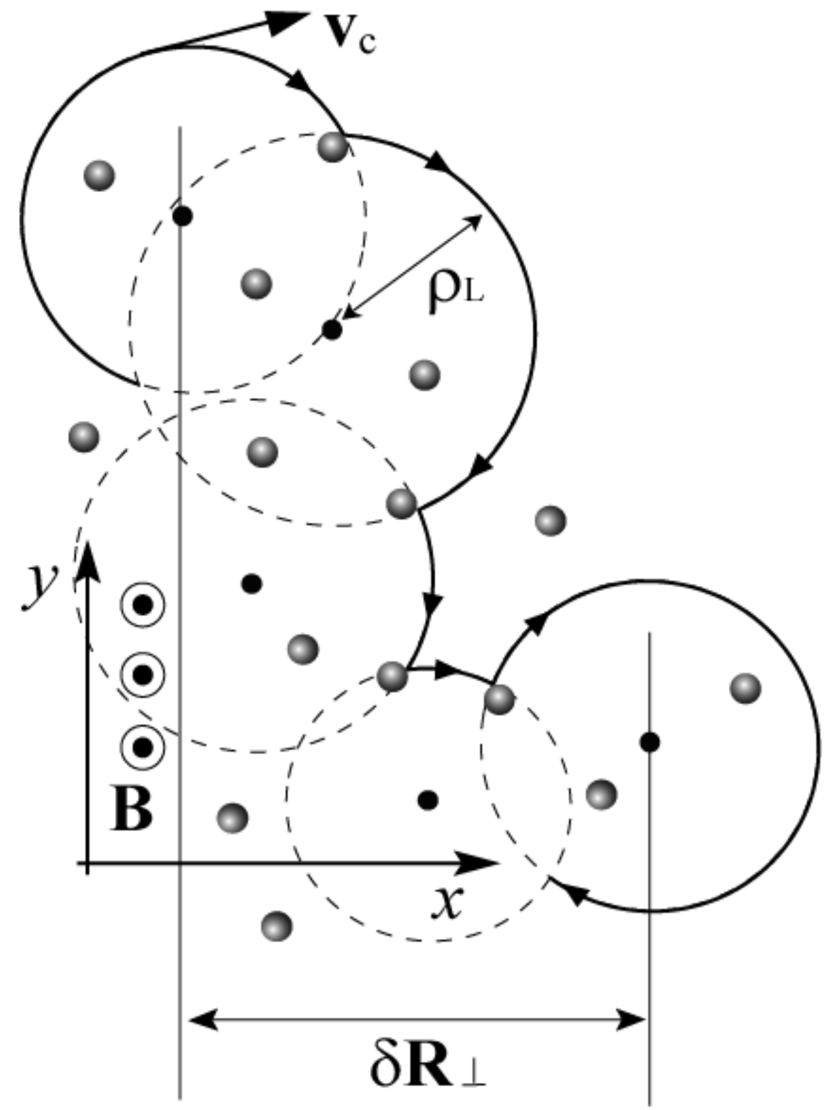
$$\delta \mathbf{r} = 0$$

$$\delta \mathbf{R}_\perp = \frac{\delta \mathbf{v}_c \times \mathbf{b}}{\omega_c}$$



$$\delta \mathbf{R}_\perp = \frac{\delta \mathbf{v}_c \times \mathbf{b}}{\omega_c}$$

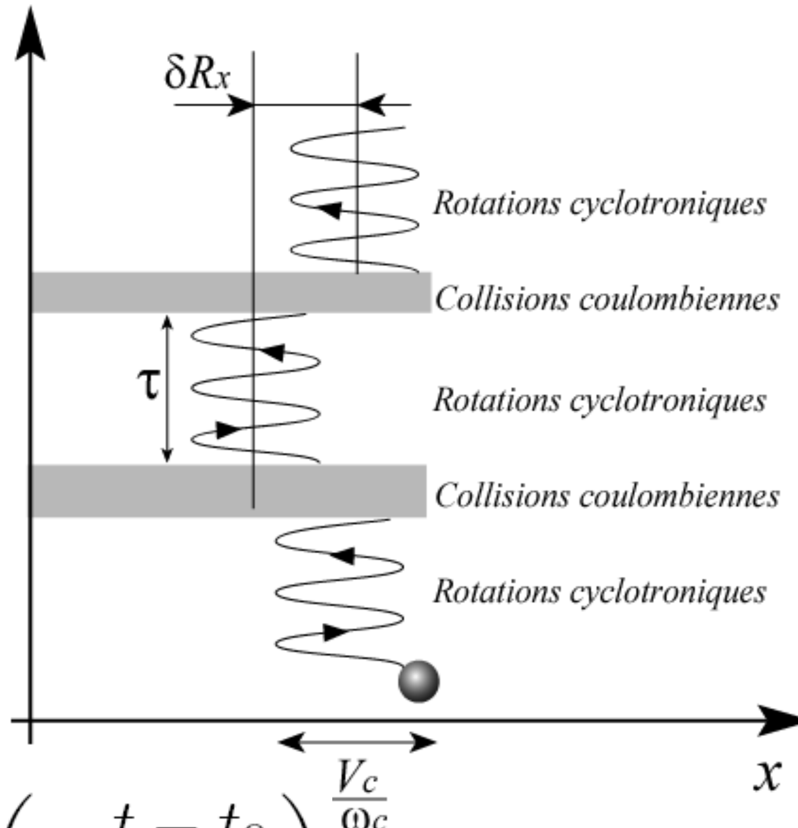
$$\langle \delta \mathbf{R}_\perp^2 \rangle = \frac{\langle \delta v_x^2 \rangle + \langle \delta v_y^2 \rangle}{\omega_c^2}$$



$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}\mathbf{v} \times \mathbf{B} + \sum_n \delta\left(t - \tau_n\right)\delta\mathbf{v}_n$$

$$\mathcal{Z} = v_x + j v_y$$

$$\frac{d\mathcal{Z}}{dt} + j\omega_c \mathcal{Z} = + \sum_n \delta\left(t - \tau_n\right)\delta\mathcal{Z}_n$$



$$\delta\mathcal{Z} = \mathbf{v}_{c0}\left[\exp\left(-j\omega_c t\right) - \exp\left(-j\omega_c t_0\right)\right]$$

$$\tau = t - t_0$$

$$\delta v_x \delta v_x + \delta v_y \delta v_y \equiv \delta\mathcal{Z}\delta\mathcal{Z}^* = 4\mathbf{v}_{c0}^2 \sin^2\left(\omega_c \frac{t - t_0}{2}\right)$$

$$\delta v_x \delta v_x + \delta v_y \delta v_y \equiv \delta \mathcal{Z} \delta \mathcal{Z}^* = 4 \mathbf{v}_{c0}^2 \sin^2\left(\omega_c \frac{t - t_0}{2}\right)$$

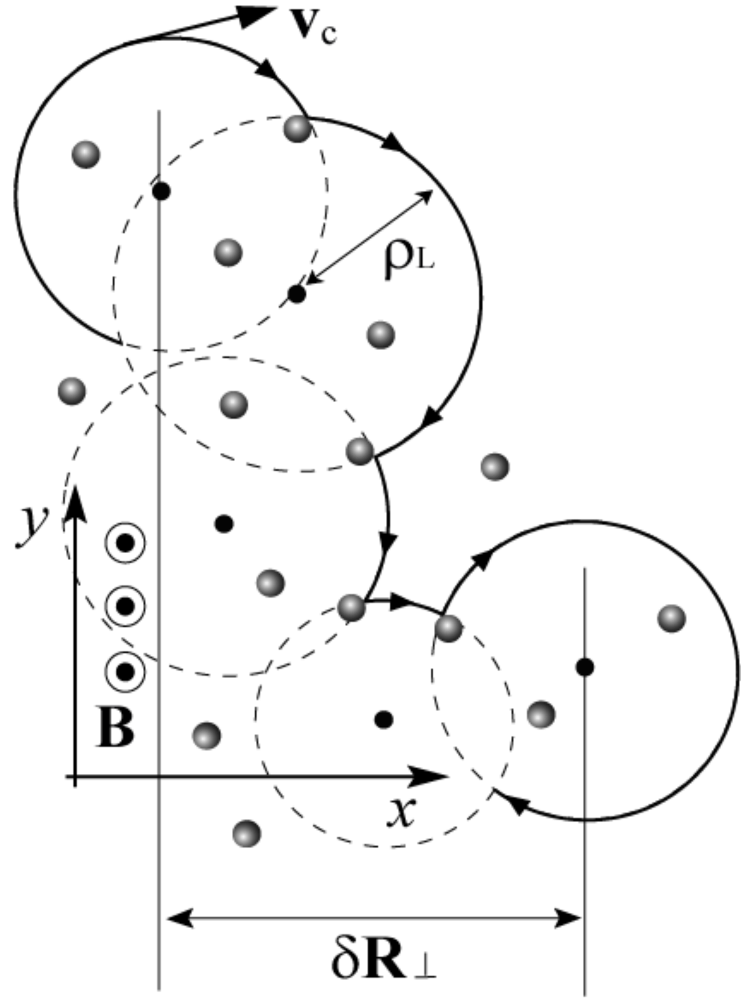
$$\left\langle \delta \mathbf{R}_\perp^2 \right\rangle = \frac{\left\langle \delta v_x^2 \right\rangle + \left\langle \delta v_y^2 \right\rangle}{\omega_c^2}$$

$$\delta \mathbf{R}_\perp \cdot \delta \mathbf{R}_\perp = 4 \frac{v_{c0}^2}{\omega_c^2} \sin^2\left(\omega_c \frac{\tau}{2}\right)$$

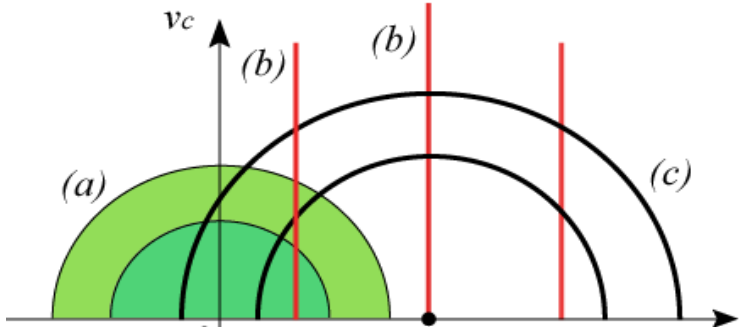
$$dP(\tau) = \nu \exp(-\nu\tau)\, d\tau$$

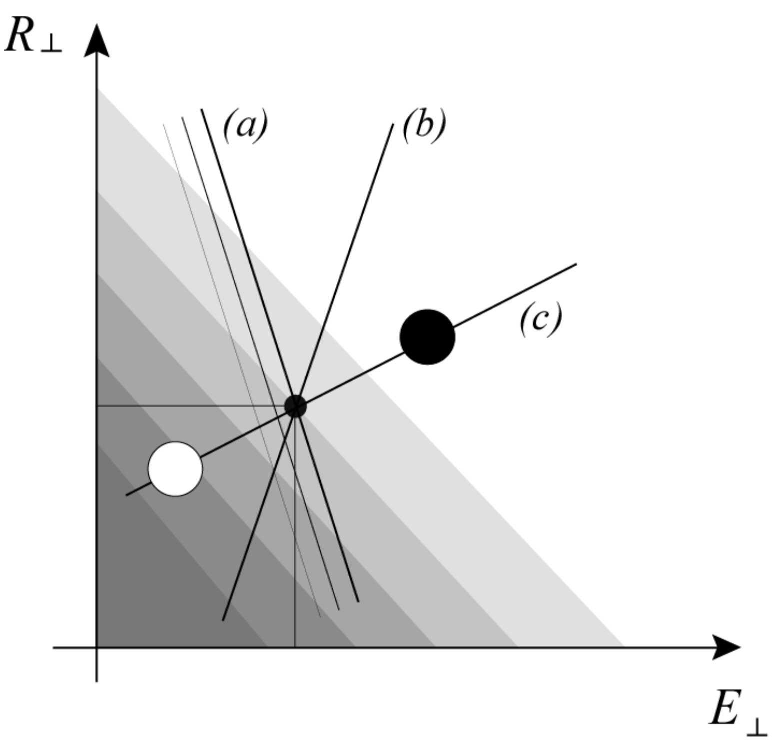
$$\frac{\left\langle \delta \mathbf{R}_\perp^2 \right\rangle}{\langle \delta t \rangle} = 4 \frac{v_{c0}^2}{\omega_c^2} \frac{\int_0^{+\infty} \exp(-\nu\tau) \sin^2\left(\omega_c \frac{\tau}{2}\right) d\tau}{\int_0^{+\infty} \tau \exp(-\nu\tau)\, d\tau}$$

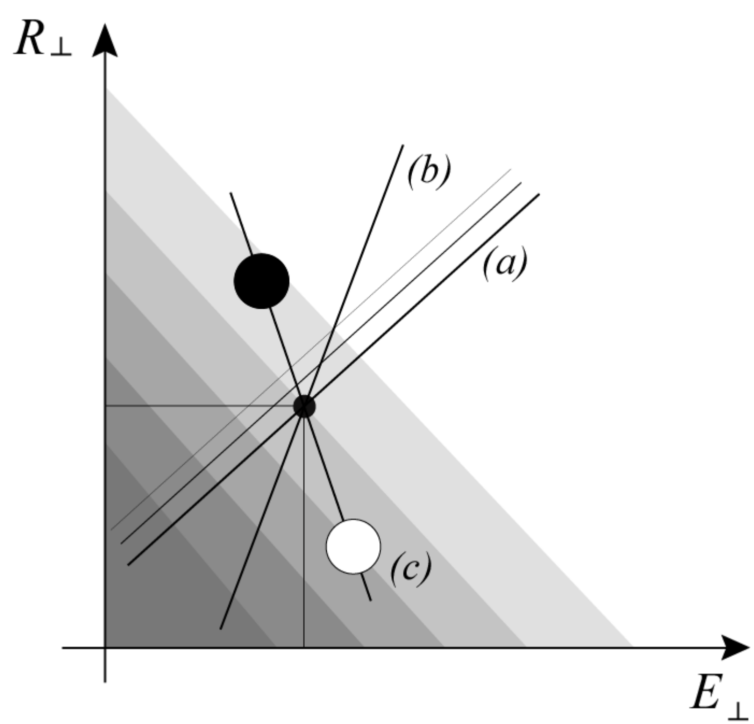
$$\frac{\left\langle \delta \mathbf{R}_\perp^2 \right\rangle}{\langle \delta t \rangle} = 2 v_c^2 \frac{\nu}{\nu^2 + \omega_c^2}$$

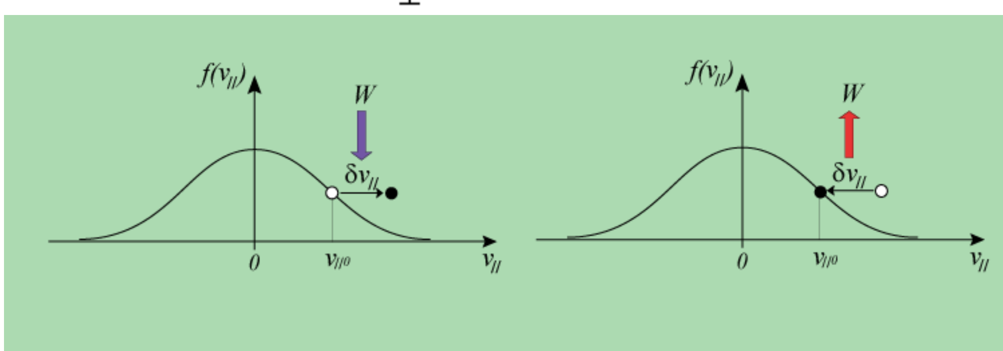


Coefficient de diffusion transverse : $$D_\perp \equiv \frac{\langle \delta R_x^2 \rangle}{2\langle \delta t \rangle} = \frac{\langle \delta R_y^2 \rangle}{2\delta t} = \frac{D_\parallel}{1+\frac{\omega_c^2}{\nu^2}}$$

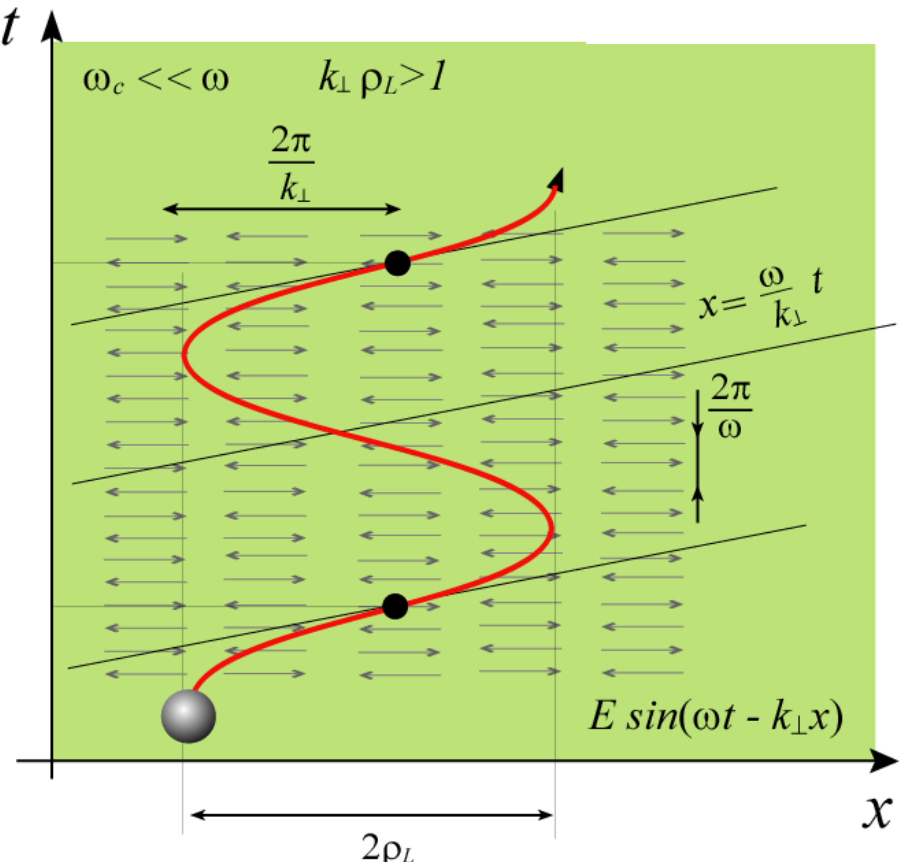

vc
(b)
(b)
(a)
(c)


R⊥
(a)
(b)
(c)
E⊥


R⊥
(b)
(a)
(c)
E⊥


f(v//)
W
δv//
0
v//0
v//
f(v//)
W
δv//
0
v//0
v//

$$\frac{\langle \delta v^2 \rangle}{\delta t} = \pi \frac{e^2}{m^2} E^2 \delta \left(kv - \omega \right) \rightarrow \quad \begin{array}{l} E \rightarrow k_\perp \phi \cos\theta \\ kv - \omega \rightarrow k_\perp v_c \cos\theta - \omega \\ \langle \delta v^2 \rangle \rightarrow \langle \delta v_c^2 \rangle \end{array}$$



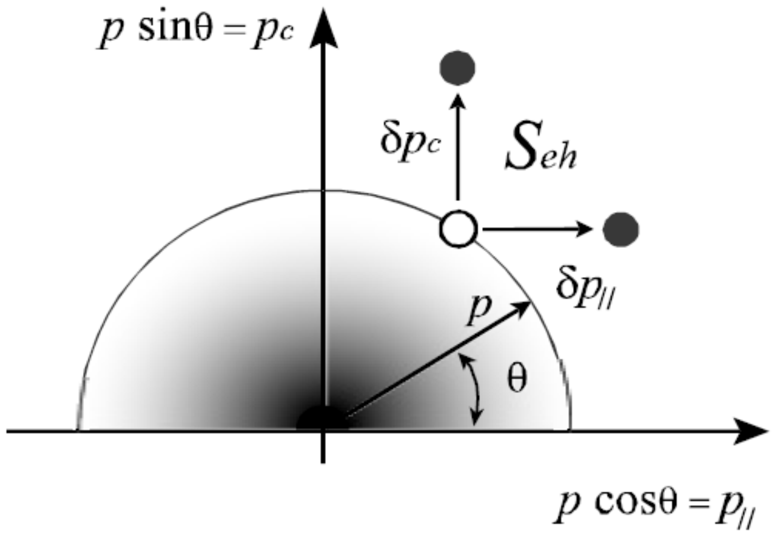
$$\frac{\langle \delta v_c^2 \rangle}{2\delta t} = \frac{\pi}{2} \frac{e^2}{m_\alpha^2} k_\perp^2 \phi^2 \cos^2\theta \delta \left(\omega - k_\perp v_c \cos\theta \right)$$

$$\left\langle \frac{\langle \delta v_c^2 \rangle}{2\delta t} \right\rangle \Bigg|_\theta = \frac{Z_\alpha^2 e^2}{2m_\alpha^2} \left(\frac{\omega}{k_\perp v_c} \right)^2 \frac{H\left(v_c - \frac{\omega}{k_\perp} \right)}{\sqrt{k_\perp^2 v_c^2 - \omega^2}} k_\perp^2 \phi^2$$

Physics of Landau and Cyclotron Resonances : Current Generation and Free Energy Extraction

- Active and reactive power
- Plasma resonances
- Resonant interaction
- Random phase approximation RPA
- Quasi linear equation
- Landau absorption
- Cyclotron absorption
- Current generation 1D
- Current generation 2D
- Free energy extraction

Transfert d' impulsion



$$\frac{dv_{\|}}{dt} = -\nu v_{\|} + \delta v_{\|}\left(v_{\|0}\right)\delta\left(t\right)$$

$$\frac{dv_{\|}}{dt} = -\nu v_{\|} + \delta v_{\|}\left(v_{\|0}\right)\delta\left(t\right) \rightarrow v_{\|}\left(t\right) = v_{\|0} + \theta\left(t\right)\delta v_{\|}\left(v_{\|0}\right)\exp -\nu t$$

I (t) [A] = eµ(t) / 2¼R 0 ±vk ³ vk0 ´ exp ¡ ºt

W (t) [W] = me vk0 ±vk ³ vk0 ´ ±(t)

$$\textit{Génération non-inductive}: \frac{I}{W} \equiv \frac{\int I\left(t\right)dt}{\int W\left(t\right)dt} = \frac{e}{2\pi R_0 m_e v_{\|0}\nu}$$

$$Suprathermique \quad : \quad \nu = \frac{n_e Z e^4 \Lambda}{4\pi\varepsilon_0^2 m_e^2 v_\parallel^3} \rightarrow \frac{I}{W}\left[\frac{\mathrm{A}}{\mathrm{W}}\right] = \frac{2\varepsilon_0^2 m_e}{R_0 e^3 Z \Lambda n_e} v_\parallel^2$$

$$Subthermique \quad : \quad \nu = \frac{n_e Z e^4 \Lambda}{3\left(2\pi\right)^{\frac{3}{2}} \varepsilon_0^2 m_e^{\frac{1}{2}} \left(k_B T_e\right)^{\frac{3}{2}}} \rightarrow \frac{I}{W}\left[\frac{\mathrm{A}}{\mathrm{W}}\right] = \frac{3\pi^{\frac{1}{2}}\varepsilon_0^2 m_e}{2R_0 e^3 Z \Lambda n_e} \frac{v_T^3}{v_\parallel}$$